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Optimal Staging Of Russian River Basin Development

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A planning model for a river basin development is constructed and applied to the Russian River Basin in California. Using dynamic programming methods, the problem is set in both deterministic and stochastic frameworks. In the latter, future population in the region and the corresponding time–dependent, price–sensitive demand relationships for water are projected with stochastic disturbance. A learning mechanism is incorporated in the algorithm so that population projections are updated with time by a Bayesian rule, and an optimal investment strategy is obtained for each time period conditional upon observed population at that period.

Application to the Russian River Basin recognizes explicitly development of the three major multipurpose water projects of the Basin. Optimal timing and scheduling of these projects strongly depends upon population growth, but ordering of their construction does not. Results indicate that it would be optimal to construct only two of these three projects within the next 50–year period—a second stage of the currently existing Lake Mendocino and the Warm Springs projects. The third and largest project (Knights Valley) should be postponed beyond the year 2020 (the planning horizon in the present study). For economically reasonable discount rates, the Lake Mendocino enlargement should precede the Warm Springs project. The economically efficient solution calls for construction of the former between 1980 and 1990. The latter should not be constructed prior to the year 2000, its optimal timing depending on population growth. Under current projections of population growth by the California Department of Finance, the optimal timing for constructing the Warm Springs project is between 2005 and some time after 2020, the end of the planning period.

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CONTENTS

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OPTIMAL STAGING OF RUSSIAN RIVER BASIN DEVELOPMENT

I. INTRODUCTION

River basin development typically involves investment decisions over time where decisions are made under conditions of partial information. The present work develops an analytical framework and generates optimal empirical solutions related to water development in the Russian River Basin (RRB) in California. This problem is viewed here as one of sequencing and timing of a finite number of projects each of which possibly could be built in a discrete and finite number of stages.

The discreteness of water development projects lends itself to treatment by the dynamic programming (DP) methodology developed by Bellman (1957) and applied by Bellman and Dreyfus (1962) to a vast range of problems. Hall and Buras (1965) were among the first to apply DP to water resources development. Butcher, Haimes, and Hall (1969) have used this framework for optimal sequencing of water supply projects. They derived a minimum present–cost solution for investment in water projects to meet a given time path of water requirements.

Development of a more efficient DP algorithm for the sequencing and scheduling of water supply projects was the major purpose of the work by Morin and Esogbue (1971) who modified the algorithm used by Butcher, Haimes, and Hall (1969) to solve a more general scheduling problem. In a recent paper, Erlenkotter (1973) has developed a DP model for sequencing hydroelectric projects which accommodates explicitly interdependencies among projects.

All of the above–cited applications of DP to water development have considered the demand for water (or hydroelectric power) as requirements which are to be satisfied, and optimization has involved minimizing the cost of meeting these requirements. A different approach to optimization is taken by Riordan (1971a) who used a more general economic efficiency criterion to obtain a solution to a more general type of pricing investment problem. In his work a price–sensitive demand relation for the output of the projects under consideration is introduced, and a marginal cost-pricing criterion is defined as required for economic efficiency. Riordan (1971b) later applied this model to an investment–pricing problem in urban water supply facilities using "typical but hypothetical cost and demand curves" (Riordan, 1971b, p. 463).

A major limitation of the above–cited applications of DP to water resource development is that uncertainty is disregarded in the underlying models, although reference to incorporation of uncertainty is made frequently as a desirable extension. Hall and Dracup (1970) discuss three methodologies for incorporating stochastic inputs into water supply decision problems: critical period analysis, expected value optimization, and Monte Carlo analysis. They conclude that the expected value optimization is appropriate where risk is negligible. Rausser and Dean (1971) survey a few possible approaches for dealing with uncertainty in water resources decision making. Bayesian methods have been used by Rausser, Willis, and Frick (1972) to accommodate the learning mechanism in an investment–decision problem related to a water desalination project. Also, price–sensitive water demand and incorporation of uncertainty and learning from a stream of information over time were important features of earlier work on the research reported here (Regev, 1967).

The purpose of this work is twofold: (1) to develop both a deterministic and stochastic DP model for the scheduling of investment in water resource projects of a river basin and (2) to apply these models to the RRB in California. In the empirical application, the deterministic model is used most extensively to obtain optimal scheduling of the major potential water projects in the Basin. In the application of the stochastic model, projected population in the area to be served by the Basin development is assumed to be the only random variable. The optimal solution in this case takes the form of a strategy, rather than a predetermined plan, conditional upon observed population growth in the service area as time passes.

Section II presents the theoretical framework within which the sequencing and timing problem is solved for both the deterministic and stochastic formulations. Section III opens with some relevant empirical descriptive background and proceeds from there to describe procedures used in developing empirical approximations for various parameters used in the empirical analysis of section IV.

Most of the results presented in section IV are derived from the deterministic model. The basic results presented there are supplemented by sets of (deterministic) results derived in sensitivity analyses to alternative empirical measures of annual recreation benefits as one component of total benefits from the projects under consideration and to alternative discount rates. The reasons for these sensitivity analyses are developed in the supporting exposition in sections III and IV. Also presented in section IV are empirical measures of opportunity costs of two suboptimal plans, recognizing the fact that construction of the Warm Springs project has already begun which represents a departure from the optimal sequencing and timing of projects implied by the basic results derived in the present study.[1] Finally, this section concludes with results derived from the stochastic model, recognizing projected population as a random variable. As noted above, the results in this case take the form of an optimal strategy conditional upon observed population growth rather than a predetermined optimal plan as in the deterministic model.

II. THEORETICAL FRAMEWORK

The problem of water development in a river basin is a special case of the more general problem of investment over time which is viewed here as an investment–decision process. The analytical solution of the problem is facilitated by looking at the problem as a dynamic system the evolution of which is characterized by (1) the state of the system at time t and (2) the decision (investment) taken at time t. The evolution of the system over time is determined by transformation rules which may be either deterministic or stochastic. A returns function is defined for each time, given the state and decision. Using a discount factor, the maximization of the present value of the stream of returns yields the solution to the problem.

In the problem at hand, the system is defined by a finite number of water projects in the basin where the state variables represent existing projects and the decision variables are investment decisions to undertake new projects or invest in increasing the size of existing projects. The role of state variables in any decision process requires that they contain all characteristics which are relevant for the decision. To simplify the problem, it is assumed

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[1] At the time of this writing, the present construction of the Warm Springs project is being contested in court and is at least temporarily halted as a result of this court action.

that, for each project (which is essentially the construction of a dam which creates a water reservoir), an expected water yield can be estimated as a "firm annual yield" which is a monotone–increasing function of the size of the dam. The state variables are defined, therefore, as the set of firm annual yields of the projects. However, this constitutes the entire set of state variables only in the deterministic framework and is augmented by additional variables––"information" variables––when a stochastic return function is considered. The decision variables are defined in a similar way; a decision to invest in a project results in an additional amount of firm annual yield of water from that project. Thus, an investment decision is defined by variables of the same dimensions as those of the state variables, namely, by firm annual yield of water provided by such decisions.

In the stochastic model the dependence of the return function on a random population growth pattern is considered as the only stochastic component in the system, while all other components are treated as if they were known with certainty. In this model the state of the system includes, in addition to the former state variables, a population state variable. The evolution of the system from one state to another is characterized by a stochastic transition function. Furthermore, observable information on population size obtained at each date is used to "improve" the estimates of the distribution function of population size which yields an investment strategy that is conditional upon this information.

We now turn to a more rigorous formulation of the problem, first in a deterministic and then in a stochastic framework.

A Deterministic Model

Define:

v_{jt} = water yield of *j*th project available at year t; $j = 1, 2, ..., n$ and $t = 0, 1, 2, ..., T$.

v_t = $[v_{1t}, ..., v_{nt}]$, a vector representation of v_{jt}.

V_t = $\sum_{j=1}^{n} v_{jt}$.

x_{jt} = water yield of *j*th project constructed at year t, to be made available at year $t + 1$.

x_t = $[x_{1t}, ..., x_{nt}]$, a vector representation of x_{jt}.

$R_t (v_t, x_t)$ = net return function at year t associated with state v_t and decision x_t.

$Z_j (x_{j1} ... x_{j,T-1})$ = terminal value of the *j*th project at the end of planning period (T).

$\beta_t = 1/(1 + r)^t$ = discount factor where r is a constant discount rate. β_1 is also denoted by $\beta = 1/(1+r)$.

The investment problem is to find a sequence $\{x_1, ..., x_t\}$ which

$$\max \left\{ \sum_{t=0}^{T} \beta_t \; R_t \; (v_t, \; x_t) \; + \; \beta_T \sum_{j=1}^{n} Z_j \; (x_{j1} \ldots x_{j,T-1}) \right\} \tag{2.1}$$

subject to

$$x_t \in X \; (v_t) \tag{2.2}$$

$$v_{t+1} = v_t + x_t, \; v_o = 0. \tag{2.3}$$

Equation (2.1) represents the present value of the stream of net returns resulting from the investment decisions over all years within the planning horizon plus a terminal value function which represents the value of the projects beyond the planning horizon. In the application of the model, $R_t \; (v_t, x_t)$ is defined by

$$R_t \; (v_t, \; x_t) = B_t \; (V_t) + \sum_{j=1}^{n} \left[D_j \; (v_{jt}) - 0_j \; (v_{jt}) - C_j \; (v_{jt}, \; x_{jt}) \right] \tag{2.4}$$

where

$D_j \; (\cdot)$ = annual flood control and recreation benefits of the *j*th project

$O_j \; (\cdot)$ = annual operation and maintenance cost of the *j*th project

and

$C_j \; (\cdot)$ = construction cost of the *j*th project.

The first term in (2.4) serves to incorporate conservation benefits, which are project independent,[1] into the objective function (2.1). The other terms in square brackets are project–related benefits and costs. A detailed discussion and parametrization of $R_t \; (\cdot)$ appear in section III. The terminal value function, $Z_j \; (X_{j1} \; ... \; x_{j,T-1})$, is intended to represent the dependence of the terminal value of the *j*th project upon the dates as well as the magnitudes of investment in it where it is assumed that the *j*th project could be either constructed at a single step or in a few stages. The importance of a terminal value function is that it allows an operational finite horizon solution to an essentially infinite horizon problem. The role of the terminal value function is to represent the value of the investment decisions up to T (the planning horizon) for "future generations." Using

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[1] Water flows within the basin by gravitation in one direction––from north to south––and, as long as the solution to the problem maintains this flow, the assumption that water conservation benefits are project independent is theoretically valid; see Regev and Schwartz (1973).

a discount factor $0 < \beta < 1$, it is clear that, the closer the planning horizon and the higher the discount factor, the more crucial is the terminal value component of the objective function in its effect on the optimal solution.[1]

The decision space, $X(v_t)$, in equation (2.2) represents all feasible investment decisions. In the current problem, each project is bounded from above by physical and hydrological conditions. Denote this upper bound by $\bar{v}_t = (\bar{v}_{1t}, ..., \bar{v}_{nt})$; then,[2]

$$X(v_t) = \left\{ x_t : 0 \leqslant x_t \leqslant \bar{v}_t - v_t \right\}. \tag{2.5}$$

Equation (2.3) relates investment decisions to the state variables in a way such that a decision to invest in a project at any date implies that the project is completed at the consecutive date. If construction of a project requires a longer period than that represented by $[t, t + 1]$, this formulation needs to be adjusted accordingly.[3]

Methods of DP are employed here to obtain the optimum investment staging solution (Bellman, 1957; Bellman and Dreyfus, 1962). This involves a stepwise solution to the maximization of (2.1). That is, instead of seeking a simultaneous solution of the $(n \times T)$ multistage problem, it is solved at each stage as an n–dimensional decision problem on $x_t \in X(v_t)$.

For purposes of describing the solution method, it is convenient to index stages in reverse order of time. Accordingly, denote a stage by $k = 0, 1, 2, ..., K$ such that $k = 0 \Leftrightarrow t = T$ and $k = K \Leftrightarrow t = 0$. Then, for a time point m years from the end of the planning horizon, define

$$S_m(v_m, x_m, x_{m-1}, \ldots, x_0) = \sum_{k=0}^{m} \beta_{m-k} R_k(v_k, x_k) \tag{2.6}$$

where R_k is as defined in (2.4) but indexed over k rather than t. S_m is the present value of the net benefit stream, starting from m years before the end of the planning period T, discounted to the time point $t = m$. S_m differs from the objective function (2.1) by the exclusion of the terminal value function.

Let

$$\hat{f}_m(v_m) = \max_{x_0, \ldots, x_m} S_m(v_m, x_m, x_{m-1}, \ldots, x_0). \tag{2.7}$$

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[1] For a discussion of the problem of infinite horizon, see Burt and Cummings (1970).

[2] Various forms of budget constraints could also be accommodated in the formulation; but, since no such constraints are recognized in the subsequent empirical analysis, they are disregarded in the present development of the analytical framework.

[3] An alternative formulation of (2.3) could state $v_t = v_{t-1} + x_t$ where the subscript t of x_t denotes then the date of completion of the project.

Equation (2.7) expresses the maximization problem from any time point m to the end of the planning horizon. For m = k and $z_j = 0$ for all j, it is the maximum of (2.1).

Employing Bellman's optimality principle (Bellman, 1957), equation (2.7) may be rewritten directly in the form of the recurrence relation

$$\hat{f}\ (v) = \max_{x_m} \left[R_m + \beta \max_{x_{m-1}, \ldots, x_0} S_{m-1}\ (v_{m-1}, x_{m-1}, \ldots, x_0) \right] \quad (2.8)$$

where $v_{m-1} = v_m + x_m$. Since m may take any k value, k may be substituted for m directly in (2.8). Doing this and substituting (2.7), we obtain

$$\hat{f}_k\ (v_k) = \max_{x_k} \left[R_k\ (\cdot) + \beta\ \hat{f}_{k-1}\ (v_{k-1}) \right] \quad (2.9)$$

where $v_{k-1} = v_k + x_k$ for all k = 0, ..., K.

For $z_j \neq 0$, the foregoing procedure cannot be applied directly to the objective function (2.1) unless it is possible to define a meaningful terminal value function that can be incorporated in (2.6) through (2.9) without destroying the Markovian property of the decision process. One approximation to terminal value of a project that can be so incorporated is one based on a proportion of construction cost of that project corresponding to the proportion of life remaining for that project at T. That is, define for project j constructed at time t*,

$$Z_{jt^*} = \frac{PL_j - (T - t^*)}{PL_j} \cdot C_j\ (v_{jt^*}, x_{jt^*}) \quad (2.10a)$$

where PL_j denotes life span of project j. The left–hand term on the right is the proportion of the life span of project j constructed at t* remaining at time T. Now, in this formulation t* may take any t value from 0 to T − 1, and (2.10a) may be written omitting the * from t*.[1]

Observing that, following a decision at time t, (x_{jt}), the terminal value of project j is known, then the terminal value of j may be deducted directly from the decision cost function $C_j\ (v_{jt}, x_{jt})$ giving

$$NC_{jt} = C_j\ (v_{jt}, x_{jt}) - \beta^{T-t}\ Z_{jt}\ (v_{jt}, x_{jt})\,. \quad (2.10b)$$

[1] The terminal value function $Z_{jt}\ (\cdot)$ should also take into account any effect of the current decision on the terminal value of former stages of the same project. Thus, for example, if a decision to enlarge a given project also extends the life span of "older" stages of the project, this should be reflected by $Z_{jt}\ (\cdot)$.

Using (2.4) and (2.10b), equation (2.1) is equivalent to

$$\max \sum_{t=0} \beta_t \ NR_t \ (x_t, \ v_t) \tag{2.11}$$

where

$$NR_t = B_t \ (V_t) + \sum_j \left[D_j \ (v_{jt}) - 0_j \ (v_{jt}) - NC_{jt} \ (v_{jt}, \ x_{jt}) \right] \tag{2.12}$$

and the recursive equation (2.9) may be rewritten as

$$f_k \ (v_k) = \max_{x_k \in X(v_k)} \left[NR_k \ (\cdot) + \beta \ f_{k-1} \ (v_{k-1}) \right]. \tag{2.13}$$

When the maximization is subject to (2.2) and (2.3), this problem is equivalent to the original problem (2.1)–(2.3). The Markovian property is preserved by the definition of terminal value adopted, and the principle of optimality can be used to solve the problem via DP methods.

A Stochastic Model

In the deterministic formulation, all components of the system described above are completely specified and known. Completely known state and decision vectors lead to a definite transformation and result in a new, completely known state vector. The same determinism has been assumed for the net benefit function. This assumption of complete information disregards one crucial factor of most decision processes in the real world——uncertainty. Uncertainty can range from no information whatsoever to almost full knowledge and can be found in any of the components of the system. The state of the system may not be completely known, the decision variables can be partly or wholly unknown, or the transformation may be uncertain. Finally, the net benefit function may be subject to unknown random effects.

One advantage of using the DP approach is that the stochastic elements in the process can be dealt with in the same way as the deterministic ones without changing the structure of the method. Since the stochastic element may involve different parts of the system, the resulting forms of the functional equations might be somewhat different. In the following it is assumed that the only stochastic element involved is in the benefit function. This assumption surely simplifies the problem; however, it yields some meaningful economic results. A similar formulation can accommodate the case where the stochastic elements involve only the cost components of the return function or both the benefit and cost components.

At the general level, the return function is modified to include a stochastic term, u_t, and an expected value objective replaces equation (2.1). In the problem at hand, the benefit function B_t $(\cdot)$ is derived from a price–sensitive demand relation for water which is composed of residential, industrial, and agricultural water demands.[1] Projections of

[1] Explicit formulation and parametrization of this function appear in section III.

residential water demand (which accounts for 75–90 percent of the total demand in the study area) are subject to random variations due to uncertain population growth. A stochastic disturbance term, u_t, is defined as the difference between the projected and actual population in the area at year t.

The general form of the benefit function $B_t(V_t)$ in equation (2.4) now becomes $B_k(V_k, u_k)$ where k denotes, as before, the stage of the process k years prior to the end of the planning period. One might assume u_k normally distributed independent of k, that is, $N(\mu, \sigma^2)$. The normality assumption can perhaps be justified by the Central Limit Theorem. However, it is more difficult to justify the time (stage) independence of μ. As time goes on, more information accumulates on population size in the area, the original projections may not materialize, and the expected deviation from the original projections may differ from zero. Thus, if one assumes $\mu_{t=0} = 0$ but allows μ to change as a result of new information, this opens the opportunity to take advantage of this information; and the resulting decision process becomes sequential conditional upon the observations on actual population as they materialize over time. In other words a "learning" mechanism is introduced into the model by which, at each time period, the parameters defining the distribution function of the random variable u_k are adjusted as a result of the additional observations available up to that time on the random variable.

A general way of incorporating information in a DP formulation of the problem is to augment the state vector by adding to it one or more information variables (Bellman, 1961). Using this approach, equation (2.13) would be rewritten as

$$f_k(v_k, I_k) = \max E \left\{ NR_k(v_k, x_k, I_k) + \beta f_{k-1}\left[v_{k-1}, \phi(I_k)\right] \right\} \quad (2.14)$$

where

E = expectation operator

I_k = vector of information variables

and

$\phi(I_k) = I_{k-1}$ = transformation rule for the information variables.

The notion of information variables, thus far vague, is now made explicit when it is replaced by the parameters of the distribution function of the random variable. The procedure adopted here, following Bellman (1961), is described in detail by Regev (1967).

Assume that u_k is $N(\mu_k, \sigma^2)$ and that the parameter μ_k is also a random variable. The information I_k will now be expressed in terms of the distribution function of μ_k. What this means is that the process is initiated with a certain knowledge about u_k and μ_k; but, as time goes on, additional values for u_k are observed, and the estimate of μ_k can be improved. The estimates of μ_k are "improved" with time in the sense that they are based on more information than those made at an earlier time. For example, an estimated $\mu_k < 0$ implies that the original projected population and the resulting benefits have been overestimated for the *k*th stage; and this leads to correction of the estimates of benefits for the future.

The information variables are introduced in the form of a distribution function of μ, employing the Bayes approach to modify the prior distribution.

Accordingly, let

$$\begin{aligned} g\,(u\,|\,\mu) &= N\,(\mu,\ \sigma^2) = \text{conditional probability distribution of u (given } \mu) \\ h_k\,(\mu) &= N\,(\alpha_k,\ \gamma_k^2) = \text{prior distribution of } \mu \text{ at } k\text{th stage.} \end{aligned} \tag{2.15}$$

Observing u at the kth stage, we wish to obtain the posterior distribution $h_k\,(\mu|u)$, which will be used in turn for state $k - 1$ as the prior distribution $h_{k-1}\,(\mu)$. Recall that k denotes the subscript for time, proceeding in reverse order. Thus, the distribution function $h_k\,(\mu)$ expresses the state of information at the kth stage, and the transformation rule for $h_k\,(\mu)$ into $h_{k-1}\,(\mu)$ is obtained on the basis of the observed value of u at the kth stage by using the Bayes method. Normal distribution functions for u and μ imply that the state of information can be expressed in terms of two parameters. This situation is accommodated by a very similar form of the functional equation, the only difference being the stochastic transformation rule.

By (2.15), the posterior distribution in the kth stage is also normal,[1] with new parameters expressed in terms of the parameters of $h_k\,(\mu)$ and the observed value of u at the kth stage. This posterior distribution, $h_1\,(\mu|u)$, becomes the prior distribution function for stage $k - 1$ so that:[2]

$$h_k\,(\mu\ |\ u) = h_{k-1}\left(u_k;\ \alpha_k,\ \sigma^2,\ \gamma_k^2\right) \text{ is } N\left(\alpha_{k-1},\ \gamma_{k-1}^2\right) \tag{2.16}$$

with

$$\alpha_{k-1} = \frac{u_k\,\gamma_k^2 + \alpha_k\,\sigma^2}{\gamma_k^2 + \sigma^2} \quad \text{and} \quad \gamma_{k-1}^2 = \frac{\sigma^2\,\gamma_k^2}{\gamma_k^2 + \sigma^2}\,. \tag{2.17}$$

The terms α_k, σ^2, and γ_k^2 to the right of the semicolon in (2.16) denote parameters. Henceforth, (2.16) will be written $h_{k-1}\,(u_k)$ or simply h_{k-1}. Note that α_{k-1} is a weighted average of u_k (current observation) and α_k (cumulative information parameter) with positive weights summing to one. Furthermore, the variance of this distribution decreases with time (decreases as k decreases) and is independent of the observed value u_k.

The expression for α_{k-1} in (2.17) shows the transformation of the information variable α from one stage to the next. The functional equation is still appropriate to deal with this problem; but now there are two random variables, u and μ, and the expected value is taken over both. Using (2.12), equation (2.14) may now be written as:[3]

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[1] Derivation of the posterior density function is shown in various standard references; see, for example, Mood and Graybill (1963, pp. 187–192).

[2] For details of the derivation of these results, see Regev (1967, Appendix B).

[3] For notational convenience, the subscript k is henceforth omitted from v_{jk}, x_{jk}, and u_k.

$$f_k(v, h_k) = \max_{0 \leq x \leq \bar{v}-v} \left\{ \mathop{E}_{\mu} \mathop{E}_{u} \left[B_k(V, u) + \sum_{j=1}^{n} [D_j(v_j) - 0_j(v_j) - NC_{jt}(v_j, x_j)] + \beta f_{k-1}[v + x, h_{k-1}(u)] \right] \right\}. \quad (2.18)$$

Writing (2.18) more explicitly in terms of g (u|μ) and $h_k(\mu)$ gives

$$f_k(v, h_k) = \max_{0 \leq x \leq \bar{v}-v} \left\{ \int_u \int_\mu B_k(V, u)\, g(u \mid \mu)\, h_k(\mu)\, du\, d\mu + \sum_{j=1}^{n} \left[D_j(v_j) - 0_j(v_j) - NC_{jt}(v_j, x_j) \right] + \beta \int_u \int_\mu f_{k-1}\left[v + x, h_{k-1}(u)\right] g(u \mid \mu)\, h_k(\mu)\, du\, d\mu \right\}. \quad (2.19)$$

In both functions B_k and f_{k-1}, μ is not directly involved but enters only through the distribution function g (u|μ). Defining the marginal distribution,

$$Q_k(u) = \int_\mu g(u \mid \mu)\, h_k(\mu)\, d\mu \quad (2.20)$$

equation (2.19) may be rewritten,

$$f_k(v, h_k) = \max_{0 \leq x \leq \bar{v}-v} \left\{ \int_u B_k(V, u)\, Q_k(u)\, du + \sum_{j=1}^{n} \left[D_j(v_j) - 0_j(v_j) - NC_{jt}(v_j, x_j) \right] + \beta \int_u f_{k-1}\left[v + x, h_{k-1}(u)\right] Q_k(u)\, du \right\}. \quad (2.21)$$

The marginal distribution $Q_k(u)$ can be shown to be $N(\alpha_k, \sigma^2 + \gamma_k^2)$.[1]

Using (2.16), the information content expressed by $h_{k-1}(u_k)$ is normally distributed and thus is completely specified by two parameters, α_{k-1} and γ_{k-1}^2 which are uniquely specified by (2.17) in terms of u_k, α_k, and γ_k^2. Furthermore, the variance, γ_{k-1}^2, of this distribution is independent of the observed u, being dependent upon σ^2 and γ_k^2 only. Moreover, knowledge of γ_k^2 and σ^2 yields immediately γ_k^2 for all k = 0, 1, ..., K − 1. Consequently, the information content is reduced in this case to one parameter, α_k. This parameter is the expected value of both distributions h_k and $Q_k(u)$. Mathematically, it is a weighted average of the difference between the observed value and the predicted value of the stochastic variable from the initial time point up to k. Thus, it can be

[1] For derivation of the marginal distribution of a multivariate normal distribution, see, for example, Anderson (1958, Chapter 2).

considered as a measure of past prediction errors, and it is used by the model to correct future predictions.

Rewriting equation (2.21) and substituting α_k for h_k,

$$f_k(v, \alpha) = \max_{0 \leqslant x \leqslant \bar{v}-v} \left\{ \int_u B_k(V, u)\, Q_k(u)\, du + \sum_{j=1}^{n} \left[D_j(v_j) - 0_j(v_j) - NC_{jt}(v_j, x_j) \right] + \beta \int_u f_{k-1}\left(v + x, \frac{u\gamma^2 + \alpha\sigma^2}{\gamma^2 + \sigma^2} \right) Q_k(u)\, du \right\} \tag{2.22}$$

$$\text{for } k = 0, 1, \ldots, K$$

where, again, for all variables and parameters (except σ^2, which is assumed constant over k) the subscript k is omitted for notational convenience.

In order to conform to the scheme of discreteness assumed for the model, a discrete approximation of $Q_k(u)$ is adopted such that

$$\int_u B_k(V, u)\, Q_k(u)\, du \cong \sum_{i=1}^{M} B_k(V, u_i) P_{i,k,\alpha},\; P_{i,k,\alpha} \geqslant 0 \text{ and } \sum_{i=1}^{M} P_{i,k,\alpha} = 1 \;\forall\; k, \alpha$$

where $P_{i,k,\alpha}$ is the probability that $u \in (u_i, u_{i+1})$ at the *k*th stage, given α. The computational form of the functional equation then becomes

$$f_k(v, \alpha) = \max_{0 \leqslant x \leqslant \bar{v}-v} \left\{ \sum_{i=1}^{m} B_k(V, u_i) P_{i,k,\alpha} + \sum_{j=1}^{n} \left[D_j(v_j) - 0_j(v_j) - NC_{jt}(v_j, x_j) \right] + \beta \sum_{i=1}^{M} f_{k-1}\left(v + x, \frac{u_i\gamma^2 + \alpha\sigma^2}{\gamma^2 + \sigma^2} \right) P_{i,k,\alpha} \right\} \tag{2.23}$$

$$\text{for } k = 0, 1, 2, \ldots, K.$$

To summarize, instead of a stochastic process with a constant distribution function over time, the latter changes with time. The new distribution function of u, $Q_k(u)$, depends upon the stage as well as upon α_k (through which the information is introduced into the model) but not upon the history of the process. This procedure brings in a systematic correction of estimated benefits that enables the decision–maker to choose strategies in a sequential way after observing data of the immediately preceding period only.

III. EMPIRICAL ANALYSIS

In this section the general features of the study area which are relevant to the decision problem are first described. Next, the demand for and supply of water in the area and the cost data for the three major water conservation projects (Knights Valley, Warm Springs, and Coyote) are considered. Finally, the procedures followed in adapting these data for application of the decision model of section II to the study area are discussed.

General Description of the Study Area[1]

The study area chosen for application of the staging model is that area which possibly may be served by the Russian River's water resources. The Russian River drains an area of some 1,484 square miles. Approximately two–thirds of the area is located in Sonoma County; the remainder is in Mendocino County except for 1 percent of the area which lies in Lake County. The drainage area is about 80 miles long and from 10 to 30 miles wide.

The principal tributaries of the Russian River are Dry Creek and Mark West Creek. Other major tributaries include its East Fork, Forsythe Creek, Big Sulphur Creek, and Maacama Creek. The climate is generally warm and dry. The rainy season lasts from November to March, with annual precipitation ranging from 32 inches to 60 inches and, at higher altitudes, up to 80 inches. Snowmelt has very little effect on flood runoff, and most floods occur between November and March.

The Russian River Basin (RRB) is the closest major river basin to San Francisco Bay from the north. The areas in close proximity to the RRB to the south, which are relatively heavily populated and subject to water deficiencies, include the Petaluma complex and Marin County.[2]

The RRB has inherent abundant water resources, ample to meet its present and forecasted future water demand. However, in view of increasing deficiencies in neighboring regions, projected water demand is not restricted to the RRB but includes, in addition, the projected water demand for the Petaluma complex and Marin County. The study area therefore includes, in addition to the RRB, these areas which are supplied by the water resources of the RRB. In terms of county lines, it includes the whole of Sonoma and Marin counties and parts of Mendocino (Redwood Valley and southward) as illustrated in Figure 1.

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[1] The description of the study area draws heavily on the Corps of Engineers: U. S. Department of the Army, Corps of Engineers (1964).

[2] The Corps of Engineers had originally included the Napa Valley and the Vallejo complex in the area to be served from the RRB. However, these areas (in Napa and Solano counties) have already contracted for 67,000 acre–feet from the North Bay Aqueduct (California Department of Water Resources, 1971b, pp. 60 and 67), and future water deficiencies are presumably expected to be met from sources other than the RRB (*ibid.*, pp. 64 and 73). These areas are therefore excluded from the RRB service area in the present research. Oral communication with the staff of the Corps of Engineers suggests that the Corps also does not anticipate the necessity to supply the Napa Valley and Vallejo from Russian River water resources, at least not during the planning period (ending in 2020) adopted in this study.

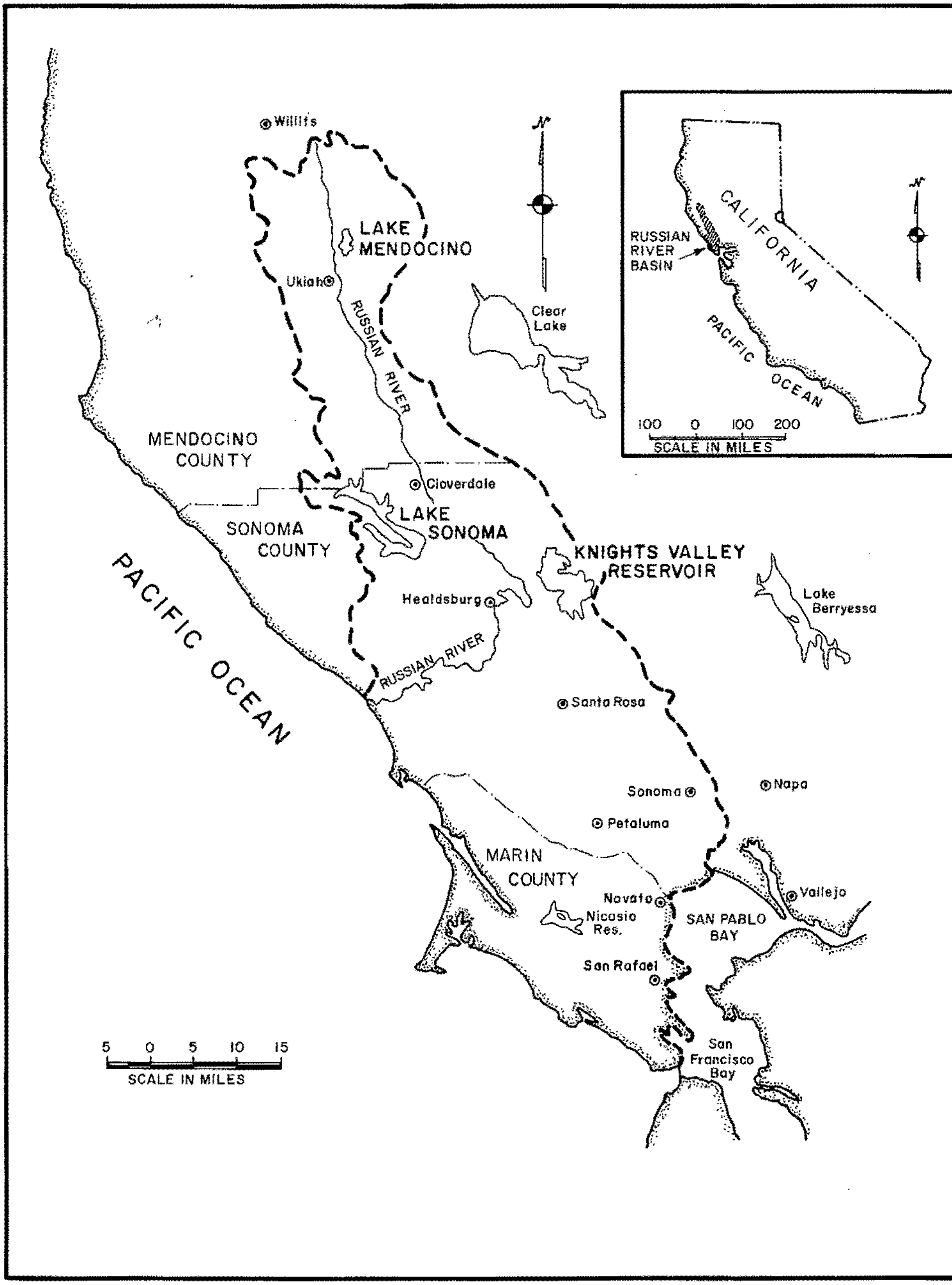

FIGURE 1. The Study Area

Projections of Future Development

Predictions of population growth in the study area are based on probable future growth of the population in the United States and of neighboring regions. Table 1 presents three population projections: (1) one made by the U. S. Department of Health, Education, and Welfare (HEW) in 1963 and used by the Corps of Engineers (later denoted as the Corps' projections); (2) one made by the California Department of Water Resources (DWR); and (3) one made by the California Department of Finance (DF) which projects a range for population in each year. The main difference in these projections lies in the assumptions used for the population rate of growth. Note that the Corps' projections were made in 1963, while the DWR projections were made in 1971, and the DF projections in 1973 (Table 1). The DWR and DF projections are, therefore, more up to date. However, the Corps' projections will also be used at certain points in the empirical analysis to examine the sensitivity of the optimal staging plan to what might now be regarded as an extreme upper limit of population growth.

Residential water demand will be projected subsequently on the basis of per capita demand for water and the above population projections.

Projections of industrial development in the RRB were made by HEW for the Corps of Engineers and used as a basis for the RRB industrial water use estimates in Table 2. No comparable projections have been made for other parts of the study area, but DWR industrial water consumption data for 1957–1959 are available on a county basis (Table 3).

Agricultural crop patterns serve as the basis for approximating agricultural demand for water. Table 4 displays the crop patterns for 1967 and those projected for 1990 and 2020, and Table 5 summarizes the agricultural water consumption by major crops. Together with the payment capacity for each crop, these data serve as a basis for projected agricultural demands for water.

Flood Damages.——The main channel along the Russian River below Ukiah is characterized by low banks and relatively shallow cross sections in some places, resulting in frequent overbank flows. Runoff occurs between October and April, with 75 percent of it occurring between December and March. Runoff closely follows rainfall, and the recession of streams is relatively rapid.

The estimated average annual damages from future floods were determined by the Corps from historical field surveys of flood losses in the RRB and reflect only tangible recurring damages. The following categories were considered in calculating historical flood damages: (1) physical damages caused by inundation, (2) emergency cost of fighting floods, and (3) business losses resulting from decreased production or increased costs of normal operations.[1] Intangible flood damages——those that cannot be given a direct monetary value——were not included in the Corps' flood damage estimates.

Average annual flood losses were computed from surveys of flood damages from 1935 to 1960 by using probability damage relationships.[2] Estimated flood control benefits,

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[1] A detailed description of types of flood losses can be found in U. S. Department of the Army, Corps of Engineers (1964, Appendix B, pp. B8–B12).

[2] *Ibid.*, pp. B12-B17.

subsequently presented, consist of anticipated damages prevented by the construction of each specific project; i.e., they constitute the difference between flood damages with and without the project.

Outdoor Recreation.—The RRB is an important recreational area in California. Mild climate and water supplies in the main river make this area highly suitable for recreation during most of the year. Water resource development cannot ignore the growing demand for recreation. This is taken into account by the Corps in its RRB development plan, both by imposing certain restrictions on minimum streamflows and minimum pool storage in reservoirs being considered for construction and by considering benefits and costs of recreation facilities associated with these reservoirs.

TABLE 1

Population Projections for the Study Area

	1960[a]	1980	2000	2020
	1,000 residents			
Corps	317	610	1,022	1,850
Department of Water Resources (DWR)[b]	317	560	930	1,330
Department of Finance (DF) maximum[c]	317	570	930	1,380
Department of Finance (DF) minimum[c]	317	545	670	760

[a] The figure for 1960 represents actual population.

[b] Data are given for 1967, 1990, and 2020. For 1980 and 2000, the numbers were obtained by linear interpolation.

[c] The DF projections are given by counties. Since the study area includes only part of Mendocino County (Redwood Valley and Russian River by the DWR terminology), the population projections for Mendocino County have been multiplied by the proportion of the population in that County within the study area (0.46), the proportion implied by the DWR projections.

Sources:

Corps: U. S. Department of the Army, Corps of Engineers (1964, Appendix D, Tables A-1, A-7, A-8, and A-9).

DWR: California Department of Water Resources (1971a; 1971b, Tables 21, 22, and 31).

DF: California Department of Finance, Population Research Unit (n.d.).

TABLE 2

Projected Russian River Basin Industrial Employment and Average Water Use

	1960	1980	2000	2020
Heavy water-using industry (1,000 employees)	1.8	3.2	4.3	5.9
Water use in 1,000 AFY[a] (2,020 GPED)[b]	4.1	7.2	9.7	13.3
Light water-using industry (1,000 employees)	7.4	16.1	24.5	40.2
Water use in 1,000 AFY (120 GPED)[b]	1.0	2.2	3.3	5.4
Services and visitors (1,000 employees and visitors)	17.1	65.2	104.7	179.3
Water use in 1,000 AFY (20 GPED)	.04	.15	.23	.40
Total urban nonresidential water use in 1,000 AFY	5.1	9.6	13.2	19.1

[a] AFY = acre-feet per year.

[b] GPED = gallons per employee per day.

Source: U. S. Department of the Army, Corps of Engineers (1964, Appendix D, Tables A-1 and A-2, Exhibits G-70, G-71, and G-72).

Recreation in the RRB is closely related to water resources since swimming, fishing, boating, and leisure living are major recreational activities in this area. However, a water development program may affect recreation in two ways: (1) by creating artificial water reservoirs, it may supply additional space for recreational activities and (2) by destroying the natural environment and by decreasing summer streamflows, it may diminish recreational activities.

The Demand for Water

The benefits that may be attributed to the water development projects here under consideration may be classified into three categories: (1) water conservation, (2) flood control, and (3) recreation. The procedure by which these benefits are incorporated into the empirical analysis will be described explicitly in a subsequent subsection. It is sufficient here to note that the measure of water conservation benefits requires prior measurement of empirical price–quantity demand relations for conservation water. That is the subject of the present subsection.

TABLE 3

Annual Industrial Freshwater Use by Counties, 1957-1959

	Marin	Mendocino	Sonoma	Total
	acre-feet per year			
Industrial water consumption	815	1,557[a]	6,813	9,185

[a] This figure includes only that part of the county in the Russian River Basin and is obtained by using the data source in Table 2.

Source: California Department of Water Resources (1964, Table 6, pp. 82-98).

Conservation water demand in the area is comprised of three basic components: residential, agricultural, and industrial and commercial demand. The procedure used is to estimate the present and projected demand for each component separately. Then total water demand for the area is obtained by summing the water quantities demanded at a given price for each component.

Residential Demand for Water

Residential water demand is determined by numerous aspects of the environment, by the price of water, and by the income of its users. In the following it is assumed that the same environmental effects on water demand prevail for the whole study area. Most statistics on urban water consumption do not differentiate residential use from commercial and industrial uses which, on theoretical grounds, may be determined by quite different price–quantity demand relations.

The water demand price elasticities found in various published works extend over a relatively wide range. Fourt (1958) has aggregated all forms of urban demand and found the elasticity of per capita demand to be around –.40. Gottlieb (1963) has estimated the price elasticity of per capita demand at between –.38 and –.67. Howe and Linaweaver (1967) have estimated linear demand relations and found for the average quantity a price elasticity of –.23. An estimate of –.31 for the price elasticity of urban demand in California is presented by Wallace (1971) together with some of the above–mentioned elasticities.

Income affects the residential demand for water mainly through bigger houses and larger surrounding areas that may require irrigation. Howe and Linaweaver (1967, p. 24) use this as the basis for measuring income effects through the proxy variable market value of the dwelling unit. Using linear demand, they obtain income elasticities in the range of .31 to .35 for the average values of the variables. Introducing income directly, Gottlieb (1963, p. 210) has obtained for the United States an income elasticity in the range of .28 to .34. Bain, Caves, and Margolis (1966, p. 189) have not found a significant relation between income and the quantity of water demanded and have explained this by the small variation in income per capita in the California cities on which their estimates are based.

TABLE 4

Current and Projected Crop Patterns in the Service Area

	1967	1990	2020
	1,000 acres		
Sonoma County			
Vineyard	2.4	8.0	17.0
Orchard	10.0	12.5	15.0
Pasture	13.0	14.9	17.8
Miscellaneous	3.5	2.5	2.0
Total	28.9	37.9	51.8
Redwood Valley and Russian River in Mendocino County			
Vineyard	.4	6.7	7.3
Orchard	4.6	6.0	6.3
Pasture	5.0	4.0	3.3
Miscellaneous	1.2	.4	.4
Total	11.2	17.1	17.3
Marin County			
Pasture	.8	.5	.4
Miscellaneous	.2	.4	.4
Total	1.0	.9	.8

Source: California Department of Water Resources (1971b, Tables 21, 24, and 33).

A linear form for the conservation water demand relation has been adopted for the present analysis. The per capita residential demand for water takes the form:

$$q_{rt} = \alpha_1 + \alpha_2 p_t + \alpha_3 y_t \quad (3.1)$$

where

q_{rt} = per capita residential water consumption in year t

p_t = water price in year t

and

y_t = per capita income in year t.

TABLE 5

Projected Agricultural Water Consumption and Payment Capacity in the Service Area

	1967	1990	2020	Payment capacity
	1,000 acre-feet per year			dollars per acre-foot
Vineyard	3.0	22.0	36.4	66.5
Orchard	19.2	37.0	42.6	41.5
Pasture	62.5	65.4	73.1	4.5
Miscellaneous	20.9	7.3	6.4	
Total	105.6	131.7	158.5	

Source: California Department of Water Resources (1971b, Tables 21, 24, and 33).

To obtain approximate empirical values for the parameters α_1, α_2, and α_3, the above estimates of price elasticity and an average price and water quantity at 1960 are used to derive a price quantity relation in which the income effect is implicitly included in the intercept of equation (3.1). The income effect on future demand projections is then incorporated additively using an estimated income elasticity and an estimated per capita income growth rate. In the procedure adopted here, a price elasticity of –.3 is imposed[1]

[1] This is within the range of the above–cited price elasticities, but it gives a heavier weight to the Howe–Linaweaver estimate which is based on linear demand estimates.

at the average price and average quantity, 38.6 cents per 1,000 gallons and 120 gallons per capita per day, respectively.[1] This gives a per capita residential demand for water at the base period (1960) of:

$$q_{ro} = 156 - .933p_o \tag{3.2}$$

where p_o is measured in cents per 1,000 gallons and q_{ro} in gallons per capita per day (GPCD).

To incorporate the income effect, an income elasticity estimate of 0.3––imposed at the average quantity of the base year––and an average income rate of growth of 2.1 percent per year––an estimate based on U. S. per capita income data for the period 1950–1970––have been adopted.[2] This leads to the following relation for projecting per capita residential water demand.[3]

$$q_{rt} = 120 + (36)\ (1.021)^t - .933p_t\ . \tag{3.3}$$

The aggregate residential demand for the study area is obtained simply by multiplying the per capita demand by projected population for the area. Doing this and at the same time transforming price and quantity to dollars per acre–foot and thousand acre–feet per year, respectively, the following aggregate residential demand relation for the area is obtained:[4]

$$Q_{rt} = \frac{[419 + (125.7)\ (1.021)^t]\ n_t}{(3117.5)\ (10^3)} - \frac{n_t}{(3117.5)\ (10^3)}\ P_t \tag{3.4}$$

which in price–dependent form becomes

$$P_t = 419 + (125.7)\ (1.021)^t - \frac{(3117.5)\ (10^3)}{n_t}\ Q_{rt} \tag{3.5}$$

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1 Average price and quantity are taken from Howe and Linaweaver (1967, p. 18, averages for 10 western areas). The average quantity conforms to the Corps estimate of 120 gallons per capita per day for the RRB; see U. S. Department of the Army, Corps of Engineers (1964, Appendix D, Table A–2).

2 U. S. President (1971, Appendix C, Table C–16, p. 215).

3 $y_t = (1.021)^t\ y_o;\ \eta_I = .3,\ q_o = 120 \Rightarrow \alpha_3 y_o = 36.$

4 For conversion, 1 acre–foot = 325,829 gallons.

where

P_t = price in dollars per acre–foot, year t

Q_{rt} = aggregate residential quantity demanded in 1,000 acre–feet, year t

and

n_t = projected area population, year t.

t = 0, 1, 2, ..., and t = 0 refers to 1960.

Finally, since approximately 55 percent of the residents in the service area resided outside the RRB in 1960, the extra conveyance cost of water is deducted from the demand price for this 55 percent in order to make it more comparable conceptually to that for Basin residents. As an estimate of this extra conveyance cost, \$40 per acre–foot was used which is the average annual operating plus (annual) construction cost of the existing Sonoma–Marin Aqueduct (California Department of Water Resources, 1973, p. vii–16). This adjustment involves subtracting (.55) (40) = 22 from the constant term in (3.5) giving:

$$P_t = 397 + (125.7)\ (1.021)^t - \frac{(3117.5)\ (10^3)}{n_t}\ Q_{rt}\ . \tag{3.6}$$

Industrial and Commercial Water Demand

Residential demand constitutes only part of the urban demand, the other part being industrial and commercial (IC) demand. Given the data available on urban water use, it was not possible to derive an empirical demand relation specifically for IC water. Lacking such data, two alternative ways of including IC demand in this analysis have been considered. The first is to assume fixed water requirements per unit (say, per employee) and to project the IC water requirement directly from projected IC development. This is done in the last row of Table 2. The second alternative is to include the IC demand together with the residential demand by imposing the parameters of the residential demand on the IC demand. The first altnerative implies an assumption of zero price elasticity for the IC water demand. Such an assumption is tenable since, typically, water cost constitutes a very small proportion of total cost in the IC sector. It is also of only minor importance in the present empirical analysis since the IC demand for water is of such minor importance in this area. On this basis, the first alternative has been adopted as the procedure for incorporating IC demand in this empirical analysis.

The estimated 9.185 thousand acre–feet of IC consumption in the service area in 1957–1959 (Table 3) is assumed to increase at a rate of 1.5 percent per year. This rate of growth is based on the Corps projections for IC employment and water consumption in the RRB (U. S. Department of the Army, Corps of Engineers, 1964, Appendix D, Exhibits G–70 and G–74). The projected time path of IC water requirements, then, is independent of water price and may be written:

$$Q_{it} = (9.185)\ (1.015)^t \tag{3.7}$$

where Q_{it} is aggregate IC water requirement, year t, in 1,000 acre–feet.

Agricultural Water Demand

The shape and form of the demand for water in agriculture are determined by the technology of agricultural production and environmental factors, such as soil, climate, etc. The relevant factors affecting the price elasticity of this demand are the technical substitutability between water and other inputs for any given crop and the degree of possible substitution among crops, both positively correlated with elasticity. Before turning to empirical evidence of the elasticity of demand for irrigation water, it is noted that most farmers in this study area own the water they use which makes statistical analysis of demand more difficult and leads toward demand elasticities higher than would be expected theoretically. Also, price schemes for irrigation water are complicated by taxes and other tolls which affect possible conclusions regarding demand elasticity.

In their empirical work on the Central Valley in California, Bain, Caves, and Margolis (1966, p. 176) concluded that demand price elasticity for irrigation water lies between –0.42 and –0.87. Moore and Hedges (1963) derived by linear programming analysis the demand for irrigation water in the San Joaquin Valley through a procedure which incorporates the optimum crop mix as a function of water price. In their analysis they report that higher demand elasticities are associated with higher prices. Over the entire range of prices, the price elasticity was estimated to be –0.65; however, for the price range of $16.50 to $30 per acre–foot, it was –0.7 and, at the average price between 0 and $16.50, it was estimated at –0.188 (Moore and Hedges, 1963, p. 20).

In this, as well as former work by Hedges and Moore (1962), essentially they assume fixed coefficients of land to water for most crops on a given piece of land. Bain, Caves, and Margolis summarized their discussion on water substitutability with the following conclusion: "On the basis of available evidence and of inferences drawn from it, we are willing to guess that on a given quality and quantity of land the substitutability of water for land within economic limits is quite poor for most crops" (Bain, Caves, and Margolis, 1966, p. 171). These conclusions strengthen the basis for the assumption of no substitutability of water for land and other inputs in the study area. The demand for irrigation water is thus obtainable from the crop patterns and their future projections.

The DWR projections of crop patterns and payment capacities have been presented in Tables 4 and 5. The estimates of payment capacities serve as proxy for the marginal value productivity of water and, together with the crop patterns in Table 5, are used to derive an empirical demand relation for irrigation water.

This agricultural demand relation is approximated by linear functions fitted by inspection to the data in Table 5 giving (directly in price–dependent form):

$$P_t = 75 - b_t Q_{at} \tag{3.8}$$

where

P_t = price (payment capacity) in dollars per acre–foot

Q_{at} = aggregate quantity demanded for agricultural use in 1,000 acre–feet

and

b_t = demand slope coefficient.

Note that the intercept in (3.8) does not vary with t, but the slope coefficient does. The value of b_t changes from 1.5 in 1960 to .75 in 2020. Empirical values of b_t for intervening years during the planning horizon are derived by linear interpolation.

Aggregate Conservation Water Demand

Aggregating the above components of conservation water demand gives an aggregate piecewise linear demand relation. Summing expressions (3.6), (3.7), and (3.8) over water quantity (Q) and transforming the resulting expressions to price–dependent form gives:

$$P_t \begin{cases} = \infty, & \text{for } 0 \leqslant Q_t \leqslant (9.185)(1.015)^t \\[2ex] = 397 + (125.7)(1.021)^t + \dfrac{(28{,}634)(10^3)}{n_t}(1.015)^t - \dfrac{(3117.5)(10^3)}{n_t} Q_t, & \\[2ex] & \text{for } (9.185)(1.015)^t < Q_t \leqslant A_t \\[2ex] = \dfrac{[397 + (125.7)(1.021)^t]\, n_t b_t + (28{,}634)(10^3)(1.015)^t\, b_t + (233{,}812)(10^3)}{n_t b_t + (3117.5)(10^3)} & \\[2ex] \quad - \dfrac{(3117.5)(10^3)\, b_t}{n_t b_t + (3117.5)(10^3)} Q_t, & \text{for } A_t < Q_t \leqslant B_t \\[2ex] = 0, & \text{for } B_t < Q_t < \infty \end{cases} \tag{3.9}$$

where

Q_t = aggregate water quantity demanded in year t (1,000 acre–feet)

$$A_t = \frac{[397 + (125.7)(1.021)^t]\, n_t + (28{,}634)(10^3)(1.015)^t - 75\, n_t}{(3117.5)(10^3)}$$

and

$$B_t = \frac{[397 + (125.7)(1.021)^t]\, n_t b_t + (28{,}634)(10^3)(1.015)^t\, b_t + (233{,}812)(10^3)}{(3117.5)(10^3)\, b_t}$$

and P_t, n_t, and b_t are as previously defined. A_t is that Q_t for which $P_t = 75$, the intercept of the agricultural water demand; and B_t corresponds to the minimum value of Q_t at which P_t becomes zero.

Water Supply and Costs of Development

Currently available water for consumptive use in the study area includes both ground- and surface water from the Russian River and its tributaries. Table 6 contains data on the presently developed water resources in the area. These resources include all presently developed groundwater, which represents more than 90 percent of the potential from groundwater resources, and the first stage of the Coyote project (Lake Mendocino) which supplies 60,000 acre-feet per year firm annual yield (U. S. Congress, House, Committee on Public Works, 1962, p. 17).[1]

TABLE 6

Presently Developed Water Resources in the Service Area

County	Ground	Surface	Total
	1,000 acre-feet		
Marin	1.0	43.0	44.0
Mendocino[a]	15.5	24.0	39.5
Sonoma	30.0	57.0	87.0
Total	46.5	124.0	170.5

[a] Redwood Valley and Russian River only.

Source: California Department of Water Resources (1971b, Tables 21, 22, and 31).

Potential water development in the RRB includes three relatively large-scale projects: the Knights Valley project, the enlargement of Coyote reservoir (Lake Mendocino), and the Warm Springs project (Lake Sonoma), which is currently under construction. The location of these projects is shown in Figure 1.

A detailed technical hydraulic description of each of the three projects appears in U. S. Department of the Army, Corps of Engineers (1964). However, a brief discussion

[1] Firm annual yield is defined as an average annual amount of water usable in consumption or production. This amount is determined for each reservoir from simulation studies based upon historical runoff at the damsite.

might be helpful here in explaining the parameters upon which the subsequent staging solutions rest. The project with the largest potential for water conservation is the Knights Valley project. This project could be built in three stages. The first (KV–I) consists basically of two dam–impounding reservoirs on Maacama Creek and Franz Creek which would result in a single combined reservoir. The next stage (KV–II) consists of a low diversion structure on the Russian River. The third (KV–III) would involve raising the dam on Maacama Creek, construction of a larger dam on Franz Creek, and increasing diversion facilities from the Russian River to the reservoir.

The Corps' estimates of natural runoff and annual water yield are based on analysis of streamflows from 1916 to 1957.[1] These surveys estimate the average natural flows of Maacama and Franz Creeks at about 76,000 acre–feet annually. A gross storage of 233,000 acre–feet would result from KV–I. KV–II would not increase this gross storage but only the firm annual yield by augmenting natural reservoir inflow with pumped Russian River water. KV–II would not increase this gross storage but only the firm annual yield by augmenting natural reservoir inflow with pumped Russian River water. KV–III would result in a reservoir with a gross storage of 1.5 million acre–feet. These stages would lead to firm annual yields of 45,000, 154,000, and 350,000 acre–feet, respectively.

The Coyote project involves raising the capacity of an already existing reservoir on Lake Mendocino which would increase the firm annual yield of the reservoir by 75,000 acre–feet. The current Warm Springs project on Dry Creek would result in a firm annual yield of 115,000 acre–feet.

Distribution throughout the basin of water supplied by the projects under consideration is planned via increased releases into and diversions from the river stream. The effect of any of the three projects on the stream level below would be to decrease flows during flood periods and increase flows during summer and fall months. This latter effect is, however, offset by increased diversions so that summer and fall stream levels may not increase significantly above current levels.[2] This distribution system raises a question of the effect of the projects being considered on groundwater usable storage. The geological conditions in the basin are such that groundwater along the Russian River is hydraulically interconnected with surface flows; thus, a change in seasonal pattern of river channel flows could result in net changes in groundwater recharge (U. S. Department of the Army, Corps of Engineers, 1964, Appendix B, p. B–27). On the other hand, the decrease in flows during the winter months caused by the impounding of water in the projects' reservoirs is not likely to affect the groundwater recharge. And, as noted above, the summer and fall stream levels may not be altered significantly.

The type of data available render the determination of *usable* groundwater storage extremely difficult, although estimates of gross groundwater storage capacity are available (U. S. Department of the Army, Corps of Engineers, 1964, p. 45). Mainly for this reason, the effect of the projects considered on groundwater has been disregarded, although the presumption that the level of water flows in the summer and fall would not be greatly affected gives some basis for believing that this omission is perhaps not of major importance.

Project costs are broken into construction costs and annual costs. The annual costs considered here include only operational, maintenance, and major replacement (OMR); interest and depreciation costs are excluded. To include the latter, together with construction costs, would amount to double counting. The relevant data on project yield and costs are summarized in Table 7.

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[1] A detailed survey and analysis of the hydrologic and hydraulic data of the RRB appear in U. S. Department of the Army, Corps of Engineers (1964, Appendix A).

[2] For data on stream flows below the Warm Springs project with and without the project, see *idem*, (1973, Figures 4 and 5, pp. 87 and 88).

TABLE 7

Water Yield and Costs for Projects Under Consideration

	Coyote project	Warm Springs project	Knights Valley project		
			Stage I	Stages I + II	Stages I + II + III
	1	2	3	4	5
Firm annual yield (acre-feet)	75,000	115,000	45,000	154,000	350,000
Construction costs[a] (thousand dollars)	9,800	74,400	74,000	118,000	199,000
Annuals costs[a] (thousand dollars)	62	630	603	1,689	3,409

[a] Includes costs to facilitate recreation development.

Sources:

Col. 1: U. S. Department of the Army, Corps of Engineers (1964, Appendix C, Tables C-1 and C-2).

Col. 2: *Idem* (1967a, Attachment F, pp. 11, 15, and 17) .

Cols. 3, 4, and 5: *Idem* (1964, pp. 51 and 52, 58, and 61).

Benefits from Project Development

As noted previously, the benefits that stem from the projects at hand can be classified into three categories: water conservation, flood control, and recreation. A preliminary investigation by the Corps has shown that production of electrical power by any water project in this area is economically inferior to other sources of energy.[1]

Conservation Benefits

Conservation benefits were measured by the Corps by the ". . . cost of producing a water supply equivalent to that which will be produced by the multiple–purpose project with a single–purpose water supply reservoir at the same site" (U. S. Department of the Army, Corps of Engineers, 1964, p. 62). Such a measure of benefits overlooks the possibility of supplying conservation demand by alternative projects at different sites. Moreover, on theoretical grounds, it should be added that the overall benefit–cost ratio used by the Corps is altogether a wrong criterion from an economic point of view when the decision problem involves a choice of alternative projects.

The measure of conservation benefits adopted in this study is the value of conservation water evaluated at the "competitive" price. When applied to the current problem, this implies that the annual (gross) benefit for a particular year from an addition of any combination of projects to the water system in the Basin is the quantity of water thereby made available multiplied by the demand price associated with this quantity on the projected empirical demand relation for that year. The total net conservation benefit from a project built in time t will be the present value of the stream of annual gross benefits minus the present value of the cost stream associated with this project. A criterion function compatible with this definition of net benefits is set out explicitly in a subsequent subsection.

Flood Control Benefits

A major purpose of two of the projects, Warm Springs and Knights Valley, is flood control. The flood control benefit of a project is the expected value of reduction in flood damages due to the construction of the project and operational policy of the resulting reservoir. This calculation is based on estimates of flood frequency and magnitude and the damages inflicted by the floods. The expected value of flood losses in the absence of the project minus the expected value of flood losses given the project and its operational policy constitute flood control benefits attributed to the project. This measure, however reasonable, leaves a wide range for speculation since the existence of a flood control project may enhance the land value of the flood plain area. This value may further depend on institutional arrangements for flood compensation.

A detailed discussion of flood control benefits on a theoretical level as well as empirical data for the three projects under consideration in the RRB can be found in Qualls (1968, pp. 63–95). In his work, Qualls points out some difficulties in the measure of "land enhancement" and suggests a model which makes this calculation unnecessary. His estimates of flood control benefits are, however, higher than those suggested by the Corps.[2]

[1] It should be noted that the present energy situation might call for a new investigation of this alternative; however, there are no data available now on this alternative.

[2] Qualls (1968, p. 87) estimated the expected annual flood control benefits for the Knights Valley project at $101,000 compared with the Corps estimate of $88,000.

Average annual flood damages in the area affected by the Knights Valley project were estimated by the Corps at $389,000 at 1960 price level and conditions (U. S. Department of the Army, Corps of Engineers, 1964, Appendix B, pp. B–17 and B–19). This figure is reduced by an estimated $88,000 as the effect of the Knights Valley project (U. S. Department of the Army, Corps of Engineers, 1964, p. 61). This latter figure is thus the flood control benefits attributed to that project. In a similar way flood control benefits for the Warm Springs project were estimated at $2,000,000 at 1967 price level and conditions (U. S. Department of the Army, Corps of Engineers, 1967b, p. 53). No flood benefits are attributed to the third project, Coyote. The Corps' measures of annual flood control benefits have been adopted and adjusted to 1970 price levels in the objective function defined subsequently. The unadjusted annual flood benefits are summarized in Table 8.

TABLE 8

Corps Estimates of Annual Recreation and Flood Control Benefits

	Coyote	Warm Springs	Knights Valley
	1,000 dollars		
Flood control benefits	0	2,000	88
Recreation benefits	0	1,450	1,178

Sources:

U. S. Department of the Army, Corps of Engineers (1967b, pp. 53 and 54).

Idem (1964, Appendix B, pp. B-19 and B-39).

Recreation Benefits

The procedure adopted by the Corps for estimating recreation benefits of a project was to project the number of visitor days at the reservoir created by the project (including general, fish, and wildlife recreation) and multiply it by $1.00 per visitor day. In this way they arrived at the estimate of $1.18 million average annual recreation benefits for the Knights Valley project (U. S. Department of the Army, Corps of Engineers, 1964, Appendix B, p. B–39) based on their projections of visitor days at the damsite.

Using a similar procedure, the Corps estimates recreation benefits for the Warm Springs reservoir at $1,450,000 annually ". . . resulting from the accelerated demand growth curve from the approved Preliminary Master Plan; 3–1/8 percent interest rate and 100–year economic period" (U. S. Department of the Army, Corps of Engineers, 1964, p. 54). No recreational benefits are attributed to the Coyote project since it is an enlargement of a currently existing reservoir. However, some costs are involved in relocation of existing recreation facilities. Recreation benefits for the three projects, as evaluated by the Corps, are summarized in Table 8.

The recreational value of water projects cannot be measured simply by the number of visitor days at the site of the project. Theoretically, the value of a visitor day should be measured by an estimated demand (competitive) price which takes account of existing recreation alternatives. The difficulties involved in an operational definition and the measurement problems of recreational value have attracted considerable attention in the literature.[1] A major weaknes of the measure adopted by the Corps is that it disregards certain opportunity costs of the recreational development proposed.

The costs not considered by the Corps are those stemming from diminishing the recreational value of the area by destroying the natural environment and wildlife in the area. There are views that this in itself outweighs any possible recreational benefits from the projects. Also, the enhancement of water–based recreation facilities made available by the projects under consideration has an alternative cost of diminishing the value of benefits from similar recreation facilities within the study area and in nearby areas.

For lack of alternative measures, the Corps' measures of annual recreation benefits have been adopted in our empirical analysis. However, the sensitivity of optimal staging to a range of annual recreation benefits is also examined. It turns out that the optimal solution is insensitive to a relatively wide range of assumptions regarding recreation benefits which lends some support to the conclusion that measurement of these benefits is not critical to the major results of this research.

The Objective Function

The objective of future water development in the RRB is approached here from the point of view of economic efficiency––that is, a planning horizon is defined, and the order and timing of projects compatible with maximum efficiency are determined.

The Corps' empirical measures of annual flood benefits, annual recreation benefits, project construction costs, and annual OMR costs are adopted here. These benefits and costs are identified specifically with individual projects. In the criterion function defined below, the annual flood and recreation benefits may be regarded as negative annual costs as they are, in effect, subtracted from annual OMR costs. Conservation benefits, on the other hand, are not identified with specific projects. The conservation water demand relation is aggregated over the entire service area. Moreover, the conservation water price, which represents the gross benefit of one unit of water at the margin, will fluctuate over the planning period in response to demand growth and increases in water supply as additional projects are developed. The marginal criterion for economic efficiency in the staging of development of conservation water in this formulation is competitive demand price equals marginal cost where total annual cost is measured net of annual flood and recreation benefits. Conceptually, the objective function achieves this result.

The objective function does involve two issues which have not been discussed specifically in respect to this empirical analysis: the discount rate and the planning horizon for which this function is to be defined.

The choice of discount rate may determine the extent and timing of development, where a higher discount rate may lead to postponement of development and, hence, to

[1] For example, see Dorfman (1965) and Merewitz (1966).

less development overall within the planning horizon. The principles and standards for planning set forth by the Water Resources Council suggest that:

> "The discount rate will be established in accordance with the concept that the government's investment decisions . . . shall be based upon the estimated cost of Federal borrowing. . . . The discount rate to be used in plan formulation and evaluation during the remainder of the fiscal year 1974 shall be 6–7/8 percent" (National Archives of the United States, 1973, p. 24822).

Following this guideline, a discount rate of 6–7/8 percent is adopted in the basic empirical analysis subsequently reported.

The planning horizon for the present empirical problem is determined primarily by the availability of data on population projections which constitute the major basis for the projection of benefits. The year 2020 is chosen for this reason as the terminal year of planning horizon. Since the planning horizon is shorter than what may be regarded as a reasonable expected life span of the projects, it is essential to include a terminal value for each project. Following the Corps, a project life for each project of 100 years is adopted here (i.e., $PL_j = 100$, for all j, in equation 2.10a), and a terminal value is estimated for each project by equation (2.10a). This procedure assumes a linear depreciation rate and assigns only the depreciated portion of construction cost to each project during the planning period. Implicitly, the procedure assumes a net benefit–construction cost ratio of one for all projects combined for their collective remaining life beyond the planning horizon.

Expression (3.10) represents an explicit (symbolic) statement of the criterion function from which the empirical results are derived.

$$\text{Max} \sum_{t=0}^{T} \beta_t \left\{ \int_{Q_0}^{Q_0+V_t} P(s)\, ds + \sum_{j=1} \left[D_{rj}(v_{jt}) + D_{fj}(v_{jt}) \right] - \sum_{j=1}^{3} 0_j(v_{jt}) - \sum_{j=1}^{3} NC_{jt}(v_{jt}, x_{jt}) \right\} \tag{3.10}$$

where

$t = 0$ at 1970

$t = T$ at 2020

$\beta_t = \left(\frac{1}{1+r} \right)^t$, where r = 6–7/8 percent

s = variable of integration

$P(Q_t)$ = aggregate conservation water demand relation at year t (expression A.4, with $Q_t = Q_o + V_t$)

$V_t = \sum_{j=1}^{3} v_{jt}$, conservation water available at year t from projects constructed prior to t (Appendix Table A–1, column 1)

Q_o = water available at t = 0 (Table 6)

$D_{rj}(\cdot)$ = annual recreation benefits from the *j*th project (Appendix Table A–2, column 2)

$D_{fj}(\cdot)$ = annual flood control benefits from the *j*th project (Appendix Table A–2, column 1)

$O_j(\cdot)$ = annual costs of *j*th project (Appendix Table A–1, column 3)

and

$NC_{jt}(\cdot)$ = net construction cost of *j*th project (derived by applying equations (2.10a) and (2.10b) to gross construction costs from Appendix Table A–1, column 2).

In the preceding pages, empirical measures of the various parameters entering (3.10) have been presented and discussed. The difficulty remains that the several measures of benefits, costs, prices, etc., that are evaluated in monetary units are, in fact, measured in terms of the price levels of different years. In particular, the years 1960, 1963, and 1967 are cited as base years for various parameters. The parameters identified in (3.10), however, have been brought to a common base in 1970. This involves adjusting various benefits and costs and the conservation water demand relations by appropriate price level factors to bring all measures to 1970 price levels. Details of the adjustments made to accomplish this are described in the Appendix. Also, 1970 was adopted as the base for measuring present value of the criterion function and, subsequently, for measuring present values of benefit and cost streams. Finally, the empirical counterpart of the symbols in (3.10) has been identified insofar as possible by referring the reader to the appropriate table or empirical equation number. This formulation is used in obtaining both deterministic and stochastic results which are presented and discussed in section IV.

A Stochastic Version of the Problem

Many components of the system are uncertain and could be regarded as stochastic variables. However, only one factor has been so recognized here, namely, the projected population in the service area. Full knowledge is assumed for all other components of the system.

Accordingly, the stochastic variable u_k is defined as the difference between observed and initially projected population at the *k*th stage. The information parameter α_k in the posterior distribution at the k + 1st stage is then the expected value of the deviation of actual from projected population. α_k is also the expected value in the prior distribution in the *k*th stage.[1]

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[1] *Supra,* p. 9.

In updating the information parameter α_k via the Bayesian procedure, the variance plays an important role in weighting new and old information. In the theoretical section it has been shown that updating the variance is independent of the observed value of the random variable u_k (equationn 2.17). The variance in any specific time period depends only on its initial value and the elapsed time from the beginning of the process. On the other hand, the information parameter α_k is a weighted average of α_{k+1} and the difference between observed and projected population at stage k + 1, u_{k+1}, with the two components of the variance (σ^2 and γ_k^2) as weights. Thus, the estimates of $\sigma^2 + \gamma_k^2$ affect the results in two ways: (1) the larger $\sigma^2 + \gamma_k^2$, the higher the probabilities associated with extreme values of benefits in equation (2.23); and (2) a higher γ_k^2/σ^2 implies a larger weight on new information and, therefore, possibly more fluctuation in the information parameter α_k.

Data available are not adequate to even approximate the variance components σ^2 and γ_k^2. However, the maximum and minimum population projections supplied by the DF do provide a range of projections which can serve as a basis for at least approximating the variance of the marginal distribution Q(u). The range given for the DF projections is based first on a "likely" range of population growth in the United States and from this an estimated range for California and for the RRB service area. This range is used to estimate the variance of projected population in the following way. The range of projected population for the study area in 2020 is from 760,000 to 1,380,000. The stochastic variable in the model (u) is the deviation of observed from the initially predicted population; and, adopting the average prediction of the DF (as of 1970) for the year 2020 (570,000), $u \in [-310{,}000, +310{,}000]$. This may be regarded as a range of u which is relevant to projected population for 2020 given the information available at 1970. Having assumed that u_k is normally distributed and assuming further that the above range covers approximately 95 percent of its probability density function, an estimate of the variance for the distribution function, $Q_k(u)$, can be obtained where this range measures approximately ±2 standard deviations. Following the discussion in section II,[1] this implies $2\sqrt{\sigma^2 + \gamma_K^2} = 310{,}000$ and $\sqrt{\sigma^2 + \gamma_K^2} = 155{,}000$. Accordingly, an approximation to the variance of the marginal distribution, $Q_K(u)$, is $\sigma^2 + \gamma_K^2 = 24{,}025 \cdot 10^6$.

As shown in section II, the variance $\sigma^2 + \gamma_k^2$ decreases with time (i.e., as k decreases) due to the decrease in γ_k^2 as additional observations materialize on the random variable u_k. This variance approaches σ^2 as the end of the planning period (2020) is approached because of the accumulated observations on actual population that become available as time passes. As noted previously, information available does not provide an adequate basis for approximating the separate components of the initial variance, σ^2 and γ_K^2. However, there is basis for regarding σ^2 as the dominant component of the variance $\sigma^2 + \gamma_k^2$ after only a relatively short passage of time into the planning period. To show this, define

$$\epsilon_K = \frac{\gamma_K^2}{\sigma^2 + \gamma_K^2}$$

and specifying $\gamma_K^2 > 0$ and $\sigma^2 > 0$, it follows that $0 < \epsilon_K < 1$. Now it can be seen from equation (2.17) that, no matter how close to unity ϵ_K is, $\gamma_{K-1}^2 < \sigma^2$. More generally, $\gamma_{K-s}^2 < \sigma^2/s$, for s = 1, ..., K − 1. Of course, for given σ^2, values of ϵ_K closer to zero mean faster convergence of $\sigma^2 + \gamma_{K-s}^2$ to σ^2; i.e., σ^2 more heavily dominates $\sigma^2 + \gamma_{K-s}^2$ for smaller values of s. On the other hand, for a given value of $\sigma^2 + \gamma_K^2$

[1] *Supra*, p. 10.

and for given ϵ_K, each succeeding value of the total variance $\sigma^2 + \gamma^2_{K-s}$, for all s, will be smaller the smaller is σ^2.

The foregoing discussion implies that the critical component of the variance $\sigma^2 + \gamma^2_K$ is σ^2; and although information is lacking for approximating σ^2 empirically, upper and lower bounds can be established based on the above approximation of $\sigma^2 + \gamma^2_K = 24{,}025 \cdot 10^6$. This approximation, taking account of the specification $0 < \epsilon_K < 1$, implies an upper bound for σ^2 just short of $24{,}025 \cdot 10^6$ and a lower bound just above zero. Upper and lower bounds of $20{,}000 \cdot 10^6$ and $1{,}000 \cdot 10^6$, respectively, have been adopted in this study. An investment strategy corresponding to each boundary value of σ^2 is presented in section IV.

The objective function in this application of the stochastic model is essentially equation (2.23) of section II which is the present value of the expected net benefits stream from the projects under consideration. The expected annual conservation benefits component of the objective function, $\Sigma_i B_k (V, u_i) P_{i,k,\alpha}$, takes the explicit form

$$\sum_i \int_{Q_o}^{Q_k} P\ (Q,\ n_k + u_i)\ dQ\ P_{i,k,\alpha}$$

where $n_k + u_i$ is observed population in the *k*th stage and the function P (Q, n) is conservation demand for water (replacing equation A.4 in the Appendix). Empirical measures of project–related annual benefits and all costs are the same as those specified for the deterministic models.

In the actual computation, only five–year periods are considered and u_k is replaced by

$$\bar{u}_k = \frac{1}{5} \sum_{i=k-4}^{k} u_i$$

with

$$Q_k\ (\bar{u}) \sim N\left(\alpha_k,\ \sigma^2/5 + \gamma_k^2\right),$$

and

$$\alpha_{k-5} = \frac{\bar{u}_k\ \gamma_k^2 + \alpha_k\ \sigma^2/5}{\gamma_k^2 + \sigma^2/5}, \quad \gamma^2_{k-5} = \frac{\sigma^2\ \gamma_k^2/5}{\gamma_k^2 + \sigma^2/5}, \tag{3.11}$$

which are obtained by replacing u_k by $\bar{u}_k$ in the derivation of (2.16) and (2.17).

IV. RESULTS: ANALYSIS AND POLICY IMPLICATIONS

This section deals with two sets of results, one obtained under deterministic population projections and the other obtained under the assumption that future population is a stochastic variable (the only stochastic element in the system). As time goes on, observations on actual population and its development over time improve population predictions for future time through a learning process discussed in section II.

In deriving the results which follow, the objective function (3.10) is maximized for grid points only. Hence, divisibility need not be assumed for each of the projects. The basic computer programs for solving both the deterministic and the stochastic formulations have been written by Constance Cartwright, based on the formulation proposed by Bellman (1961). Each program solves the scheduling and timing of discrete independent projects with the possibility of allowing each project to be constructed in several stages ($\leqslant 7$). The programs are written in Fortran language and have been run on both the CDC 6600 and the IBM 1130 with a single disc drive. The current dimensions statement in both programs limit the number of evaluation points (number of projects times number of stages for each project) to 160. However, this was planned for the 8K memory space of the IBM 1130 and could be increased when used in larger memory computers.[1]

Deterministic Model

Presented here are three optimal water development plans for the RRB under three different population projections—Corps, DF maximum, and DF minimum. These results will be referred to henceforth as the basic results. Subsequent results will examine the sensitivity of the optimal staging plan (under the Corps and DF maximum population projections) to (1) a range of assumed levels for annual recreation benefits associated with the projects under consideration and (2) variation in the discount rate. Finally, the deterministic model is used in approximating the opportunity cost of two specific "suboptimal" staging plans relative to the optimal plan.

Basic Results

The optimal plans constituting the basic results are presented in Table 9. The empirical measures of the different types of benefits, construction and annual costs, and the discount rate used in generating these basic results are identified in the definitions of terms entering expression (3.10). These results indicate that:

1. Under no population projection should construction start before 1980. The model assumes a five-year construction period, so additional water would not be available prior to 1985 at the earliest.
2. Coyote is the first project to be constructed. Under the highest population projections (Corps), construction of this project is called for in 1980; the lowest population projections (DF minimum) call for construction of this project in 1990.
3. The Warm Springs project is called for construction within the planning horizon only under the Corps and DF maximum projections but in any case not prior to 2000. If, however, the DF minimum population projections are adopted, this project should not be constructed within the planning period (ending in 2020).

[1] The program can be obtained on request from the authors.

4. The first two stages of the Knights Valley project enter the optimal construction plan toward the end of the planning period (2010 and 2015) only under the highest (Corps) population projections. The more recent, up–to–date projections lead to postponement of Knights Valley beyond 2020.

TABLE 9

Optimal Water Development Under Different Population Projections[a]

Population projection / Year	Corps	Department of Finance	
		Maximum	Minimum
1975			
1980	Coyote (75)		
1985		Coyote (75)	
1990			Coyote (75)
1995			
2000	Warm Springs (115)		
2005		Warm Springs (115)	
2010	Knights Valley Stage I (50)		
2015	Knights Valley Stage II (100)		

[a] In these results, decisions to construct projects are allowed only at five-year intervals. The year identified with a project in this table corresponds to the year construction would be initiated. A five-year construction period is assumed in each case. Accordingly, the stream of annual benefits and annual costs would commence in the first year of the next succeeding five-year period. The figure in parentheses following each project is firm annual yield in 1,000 acre-feet.

The main thrust of these results is that, by economic efficiency criteria, the development of additional water is not called for before 1980–1985. Moreover, to determine which project should be undertaken, an overall benefit–cost ratio greater than one is a misleading criterion. By the criterion of price equal marginal cost, the Coyote project is the optimal choice for supplying this additional water. Augmenting conservation benefits by combined flood control and recreation benefits, as calculated by the Corps, is not sufficient to justify the cost of early construction of the Warm Springs project.

Sensitivity of Results to Recreation Benefits and the Discount Rate

Recreation Benefits.—Because of the questions surrounding the Corps' empirical measures of annual recreation benefits, it is instructive and helpful in appraising the basic optimal plans in Table 9 to examine the sensitivity of these basic results to a range of assumed values for this class of benefits. If the basic results are relatively insensitive to annual recreation benefits over a wide enough range, this contributes to confidence in our own results in the sense that it lends support to the conclusion that errors in the measure of recreation benefits are not a critical determinant in the optimal solution.

Results are summarized in Table 10. As indicated in the column headings, annual recreation benefits are allowed to range from –2 times to +4 times the Corps estimates for each project. Thus, results in the column headed +1 are identical to those in Table 9. Results were generated for only the Corps and DF maximum population projections. All other benefits and all costs remain the same as in the corresponding basic optimal solution in Table 9.

Comparing these results with those in Table 9 demonstrates that the optimal plan is *not* sensitive to variation in estimated annual recreation benefits over a relatively wide range. The order of projects is not affected until recreation benefits become *four* times greater than the Corps estimates used in deriving the optimal plans. The effect on timing of projects is also relatively minor. Over the range of recreation multipliers –2 to 3, timing of the first project (Coyote) is not affected under either population projection. Over this same range, the second project (Warm Springs) gets postponed (advanced) at some point as one moves in the negative (positive) direction from the unit multiplier. For the DF maximum population projections, for example, postponement of five years appears at column –2 and advancement of five years appears for columns headed 2 and 3. These are minor differences as recreation benefits vary in either direction. The relative insensitivity to variation in the negative direction provides some basis for suspecting that the optimal plans would not be affected by a reasonable adjustment of the Corps' measures of recreation benefits for "recreation costs" attributable to recreational opportunities destroyed by construction of a project.

Discount Rate.—The rate of discount used to obtain the basic results in Table 9 is 6–7/8 percent, the rate suggested by the Water Resources Council's Principles and Standards for Planning (National Archives of the United States, 1973, p. 24822). On the other hand, the Corps has used a 3–1/8 percent discount rate in the analysis justifying the decision to construct the Warm Springs project first, and construction has already begun on that project. The 6–7/8 percent rate is regarded as more appropriate from the point of view of economic efficiency. Surely the 3–1/8 percent rate is too low from an economic viewpoint, although it is presumably legally permissible.[1]

It is of some interest in these circumstances to examine the staging solution within the framework adopted here but using a 3–1/8 percent discount rate. Such results are presented in Table 11 for 3–1/8 and 6–7/8 rates as lower and upper bounds and for several intervening rates. Only the discount rate varies (within each set of population projections) in these results, all other parameters and empirical measures corresponding to those used in generating the basic results (Table 9).

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[1] Note that, as future benefits and costs are estimated at fixed prices, the discount rates used should be regarded as real rates.

TABLE 10

Sensitivity of Order and Timing of Projects to Annual Recreation Benefits[a]

Recreation benefits[b] / Year	Corps population projections							Department of Finance maximum population projections						
	−2	−1	0	1	2	3	4	−2	−1	0	1	2	3	4
	1,000 acre-feet													
1975							115							
1980	75	75	75	75	75	75								115
1985								75	75	75	75	75	75	
1990														
1995						115	50							
2000		115	115	115	115							115	115	50
2005	115					50	75		115	115	115			
2010			50	50	50			115						75
2015	150	150	100	100	100	100	100						50	

[a] The numbers in the table represent firm annual yield in 1,000 acre-feet of the projects considered: 75 for Coyote, 115 for Warm Springs, and 50 for the first stage and 100 for the second stage of the Knights Valley project.

[b] Each number in this row represents a multiplier of the annual recreation benefits. No recreation benefits are attributed to Coyote; the annual benefits of $1.686 million attributed to Warm Springs are multiplied by −2, −1, 0, 1, ..., 4; and the annual benefits of $1.494 million attributed to Knights Valley are multiplied by the same coefficients.

TABLE 11

Sensitivity of Order and Timing of Projects to the Discount Rate[a]

Year \ Discount rate	Corps population projections					Department of Finance maximum population projections				
	3-1/8	4	5	6	6-7/8	3-1/8	4	5	6	6-7/8
	1,000 acre-feet									
1975	115									
1980		75	75	75	75	115	75	75		
1985									75	75
1990										
1995	75	115					115			
2000			115	115	115	75		115		
2005	50	50							115	115
2010	100	100	50	50	50	50				
2015			100	100	100		50	50		

[a] The numbers in the table represent firm annual yield in 1,000 acre-feet of the projects considered: 75 for Coyote, 115 for Warm Springs, and 50 for the first stage and 100 for the second stage of the Knights Valley project.

Lower discount rates would be expected to lead generally to earlier staging and/or to more projects being developed within the planning horizon. This pattern is apparent on inspection of Table 11. But a more striking result in this table is the reversal of order of Coyote and Warm Springs when one compares the 6–7/8 percent and the 3–1/8 percent discount rates.[1]

This change in order of projects can be explained by the increasing weight given to recreation and flood control annual benefits as the discount rate drops. Since no such benefits are attributed to Coyote, when the discount rate is sufficiently low, these benefits for Warm Springs become large and augment the conservation benefits sufficiently to compensate for the higher construction costs of this project. As a result, the optimal solution calls for early construction of Warm Springs and postponement of the Coyote project. Under the Corps population projections, construction of Warm Springs would commence in 1975. However, the reader is reminded that the Corps population projections are unrealistically high and that the DF maximum population projections are more realistic as *maximum* projections. Under the latter projections, construction of Warm Springs should not commence before 1980 even at the discount rate of 3–1/8 percent which we have argued is too low from the point of view of economic efficiency.

Opportunity Cost of Suboptimal Staging

Recognizing that the Warm Springs project is now under construction, which departs from the optimal staging plan in Table 9, it is of some interest to evaluate the opportunity cost of this departure from optimal staging. This is done by comparing the present value of net benefits from the optimal plan with the present value of net benefits from an appropriately defined suboptimal plan. For this purpose two suboptimal plans are considered here. The first (SOP1) assumes that Warm Springs will be completed (and generating benefits) by 1980. Then the "optimal" plan from 1980 on through the planning horizon 2020 was derived, given the constraint represented by the present construction of Warm Springs. For this calculation only the DF maximum population projections are used. It turns out that the optimal plan for 1980 through 2020 in this framework calls for commencing construction of Coyote in 2005 (assumed completed by 2010). As in the corresponding optimal plan in Table 9, no stage of Knights Valley appears in the present suboptimal plan.

More formally, the calculation of net benefits for each of the rival staging plans involves the evaluation of the following expression:

$$\sum_{t=0}^{T} \beta^t \left\{ P_t(Q_t) \cdot V_t + \sum_{j=1}^{n} \left[D_{rj}(v_{jt}) + D_{fj}(v_{jt}) - 0_j(v_{jt}) - NC_{jt}(v_{jt}, x_{jt}) \right] \right\} \tag{4.1}$$

where all terms are as defined in (3.10). Regarding (4.1) as appropriate for calculating net benefits rests upon the interpretation of the implied price, $P_t(Q_t)$, at each t as the competitive price for conservation water at t. $P_t(Q_t)$ is computed for each t from the

[1] In fact, the reversal occurs between the 4 percent and 3–1/8 percent rates.

piecewise linear demand relation (A.4).[1] The empirical measures of annual recreation benefits, D_{rj}, annual flood benefits, D_{fj}, and annual costs, O_j, used in (4.1) are those identified in expression (3.10). Net construction costs, NC_j, depend on time of construction and are calculated from equation (2.10b).

The second suboptimal plan (SOP2) assumes postponement of Warm Springs construction for five years, with completion in 1985. The optimal plan from 1985 on in SOP2 is identical to that from 1980 on in SOP1. The opportunity cost of SOP2, of course, would be expected to be less than the opportunity cost of SOP1. The difference between these opportunity costs provides a measure of the "gain" from a five–year postponement of Warm Springs.

In Table 12 the implied (competitive) prices for conservation water are recorded at five–year intervals for the optimal plan (Table 9) and for the suboptimal plans.[2] For each plan the price variation by five-year intervals is only suggestive of the full price time profile under that plan. In the calculation of net benefits by (4.1), annual implied prices are used. The price behavior pattern over time results from the rate of increase in water demand over time and from project indivisibilities. That is, if water volume were constant, price would increase at a rate dictated by the rate of increase in the level of demand for water. However, construction of a project reduces the price by increasing the water volume. Hence, water price drops whenever a new project is constructed. Clearly, a greater degree of project divisibility (with similar unit cost) would result in less price fluctuation over time.[3]

The present value (at 1970) of selected components of expression (4.1) is summarized in Table 13 along with the opportunity cost of each suboptimal plan and the gain from a five–year postponement of Warm Springs. Because of the configuration of time profiles of P_t and V_t under the two suboptimal plans considered here, the present value of the stream of conservation benefits is the same in SOP1 and SOP2.[4] Conservation benefits for the suboptimal plans are, however, somewhat lower than for the optimal plan.

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[1] It is known in advance that Q_t will not fall in the region corresponding to $P_t = \infty$ for any t within the planning horizon. This is clear since $(9.185)\ (1.015)^t < Q_o$ for all t. To show this, it is sufficient that $(9.185)\ (1.015)^{50} = 19.3 < 170$.

[2] For the particular suboptimal plans considered here, SOP1 and SOP2, the time profiles of implied prices are identical except for the five years, 1980-1985.

[3] Of course, it is neither plausible nor realistic to suggest that the implied annual prices be imposed on water users since highly fluctuating "optimal" annual prices resulting from technical indivisibility of projects are, perhaps, impractical; and costs of adjustment to such price fluctuation may be very high. On the other hand, it may be argued that fluctuating prices over time to water users is essential to a well–managed water program. Price changes over time reflect relative changes in demand and supply and may be viewed as an effective device for implementing economically efficient allocation of water among users.

The purpose in calculating implied prices here is not to consider the feasibility of water pricing as an instrument in water management but rather to calculate the net benefits required for appraising the opportunity costs of different staging plans.

[4] The only years for which P_t and V_t differ in SOP1 and SOP2 are the years 1980 through 1984, but the product $(P_t) \cdot (V_t)$ is zero for both suboptimal plans during these years. That is, for SOP1, $V_t = 115$ but $P_t = 0$ during these years; while, for SOP2, $P_t > 0$ but $V_t = 0$.

TABLE 12

Conservation Water Quantities and Implied Prices for the Optimal Plan and Suboptimal Plans 1 and 2 (Under Department of Finance Maximum Population Projection)

Year	Optimal plan Q_t[a]	Optimal plan P_t	Suboptimal plan 1 Q_t[a]	Suboptimal plan 2 Q_t[a]	Suboptimal plan 1 P_t	Suboptimal plan 2 P_t
	1,000 acre-feet	dollars per acre-foot	1,000 acre-feet		dollars per acre-foot	
1970	170	0	170		0	
1975	170	0	170		0	
1980	170	10.6	285	170	0	10.6
1985	170	37.4	285		0	
1990	245	0	285		0	
1995	245	16.5	285		0	
2000	245	44.5	285		9.3	
2005	245	75.3	285		42.6	
2010	360	17.5	360		17.5	
2015	360	51.9	360		51.9	
2020	360	92.3	360		92.3	

[a] $Q_t = Q_o + V_t$, where $Q_o = 170$ is the available water supply at 1970 and V_t represents additional water made available by projects here under consideration.

Perhaps it contributes to clarifying Table 13 to first compare the optimal plan with SOP1, then compare SOP2 with SOP1. The lower conservation benefits of SOP1 as compared with the optimal plan have already been noted. But the increase in annual flood and recreation benefits much more than offsets the decline in conservation benefits, so gross benefits from SOP1 are greater than gross benefits from the optimal plan. However, the present value of combined annual and construction costs increases much more than benefits, resulting in a substantial negative present value of net benefits for SOP1. The positive net benefits foregone by not following the optimal plan when combined with the negative net benefits associated with SOP1 result in a very substantial opportunity cost (present value, $36.398 million) of adopting SOP1 in place of the optimal plan.

TABLE 13

Opportunity Costs of Suboptimal Plans 1 and 2

	Optimal plan	Suboptimal plan 1	Suboptimal plan 2
	1	2	3
	1,000 dollars		
Conservation benefits	13,017	9,355	9,355
Annual flood control and recreation benefits	2,264	29,977	20,905
Annual costs	769	5,961	4,173
Net construction costs	11,287	66,544	47,300
Net benefits (NB)[a]	3,225	-33,173	-21,213
Opportunity costs (OC)		36,398[b]	24,438[c]
Gain from 5-year postponement of Warm Springs			11,960[d]

[a] Calculated by equation (4.1).

[b] Col. 1 NB minus col. 2 NB.

[c] Col. 1 NB minus col. 3 NB.

[d] Col. 3 NB minus col. 2 NB or, equivalently, col. 2 OC minus col. 3 OC.

SOP2 differs from SOP1 only in that construction of Warm Springs is postponed by five years. In evaluating the gain from such postponement, the decrease in the sum of conservation benefits and annual benefits constitutes the "cost of postponement"; and the decrease in annual costs plus the interest at 6–7/8 percent on construction cost during the five–year postponement constitutes the "benefits from postponement." Conservation benefits are not affected by postponement in this case, but the present value of combined annual flood and recreation benefits is decreased. Both annual costs and construction cost decrease in present value, and the decrease in the latter is equivalent to the interest on construction cost during the postponement. The "gain from postponement" is simply the benefits from postponement minus the cost postponement—$11.96 million in this instance. This is, of course, identical to the difference between SOP2 and SOP1 net benefits or, equivalently, the difference between opportunity costs.

Stochastic Model Results

The assumptions underlying the empirical stochastic model were presented and discussed in section III. Since the only random element in the model is projected population, the solution—investment decisions—becomes conditional upon observed population over time. In other words, the optimal solution is obtained in the form of a strategy (rather than a plan) which gives specific thresholds in terms of minimum expected population at future time points at which investment in each project becomes optimal. For a given time profile of population growth from 1970 to any time t (stage k) and for a given initial population projection for the same period, the set $\{u_s\}$, where u_s is actual minus projected population for stage s, is observed for s = K, K − 1, ..., k + 1. Using equation (2.17), this set of observed u_s uniquely defines the parameter α_k which serves as the information state variable. The solution is obtained as a strategy which gives for each project and each stage (year) the *smallest* value of α_k for which it would be optimal to construct that project. Since α_k is the expected deviation of the actual from the initially projected population for stage k, the "expected population" will be α_k plus the initial population projection; and it is possible to present the optimal strategy as the *minimal* expected population for which it would be optimal to construct each of the projects. The initial population projections alluded to refer to the original projected population time profile for the entire planning period made in 1970. In the results presented here, the initial projection adopted is an average of the DF minimum and DF maximum projections, henceforth denoted DF average.

Figure 2 presents the optimal strategy. The three positively sloped lines represent the DF minimum, DF maximum, and DF average projections; but, as noted above, only the latter is used in the computation. The two negatively sloped lines, WW and CC, represent "strategy lines" for the Warm Springs and Coyote projects, respectively. The line for each project defines the minimal expected population at each time point for which it would be optimal to construct that project.[1] These strategy lines are interpreted as follows: If observed population at the year preceding a given year t is such that the expected population (equal to α_t plus initial projected population) is greater than or equal to the expected population on the strategy line for a given project at year t, then this project should be undertaken at year t if it has not been already constructed. Thus, the optimal strategy is to construct each project in the first year that the expected population lies on or above that project's strategy line.

Note that (a) the strategy lines do not extend beyond 2015 since no project would be constructed at 2020—the planning horizon; (b) no strategy line appears for any part of the Knights Valley project since, under the DF average initial projection, construction of this project would not be initiated within the planning period; (c) the Coyote project would be constructed first and constructed before expected population reaches levels required for construction of the Warm Springs project; and (d) for each project the expected population required to justify earlier construction is higher than that called for if construction is postponed. The range of expected population which justifies construction of the Coyote project is between 540 and 700 (1,000 residents) and is between 900 and 1,000 (1,000 residents) for the Warm Springs project. This range, however, strongly depends on the variance of projected population in the service area.

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[1] To be strictly correct, the critical minimal expected population is derived here only for years corresponding to the plotted points. The strategy line is then passed through these points.

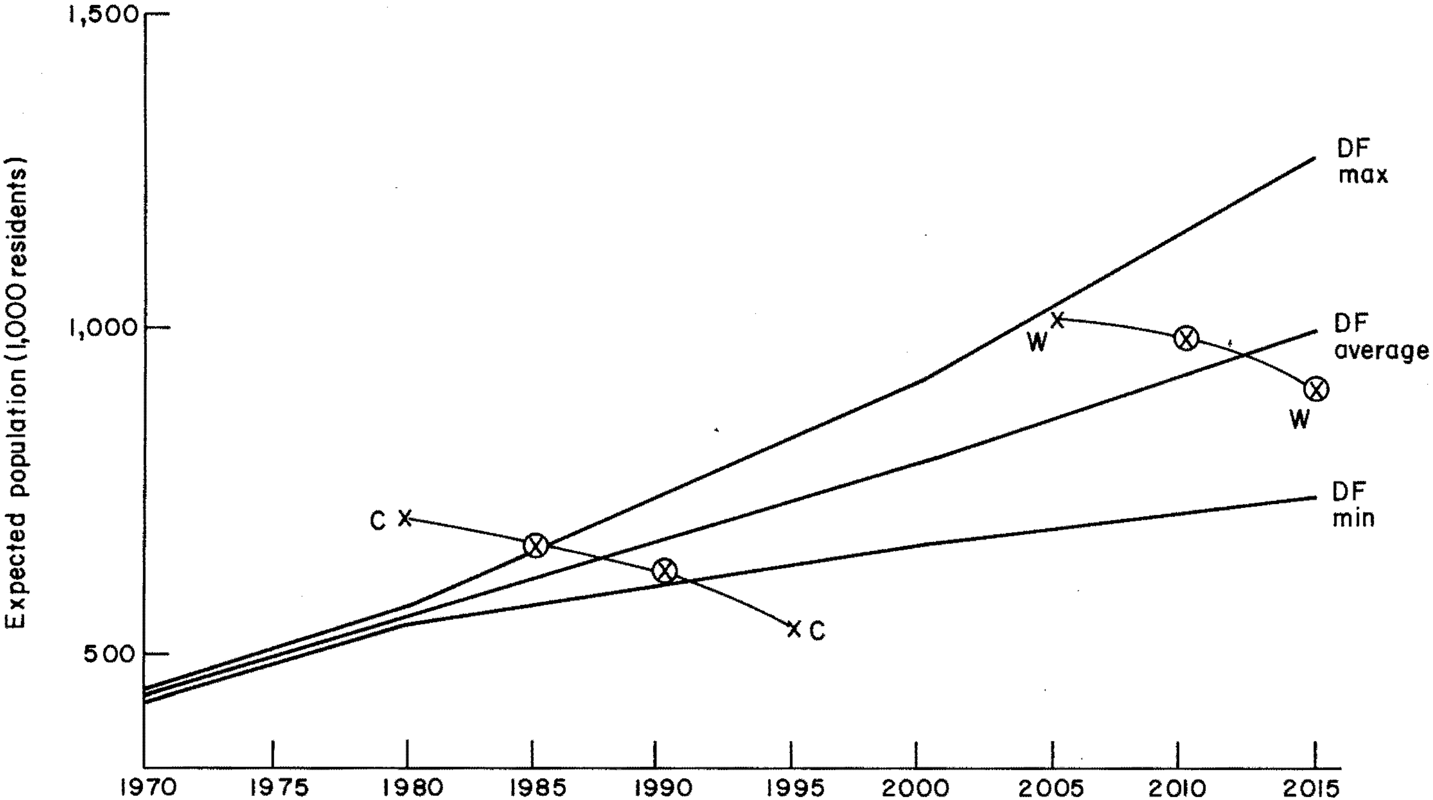

FIGURE 2. Optimal Strategy, Stochastic Model

* Solution points for $\sigma^2 = 20{,}000$ are denoted by X. Those points that apply as well for $\sigma^2 = 1{,}000$ are denoted by ⊗.

The parameter α_k plays a dual role in the computation, first, as the expected deviation of the observed population from the initially projected population at stage k and, second as a state variable summarizing the past information on population growth over that part of the planning period preceding the year represented by the kth stage. Thus, for each stage (year), the marginal probability distribution Q_k (u), which is approximated by $P_{i,k,\alpha}$ in the discrete version, can be used to obtain a transition probability matrix from a given state (α_k) to all other states at stage k − 1; and given the initial state of the system, it is possible to calculate the probability of being at any state α at stage k. This is done in the following way: Starting with $\alpha_{1970} = 0$ (by assumption), the probability distribution for $\bar{u}_k$ is given by $P_{i,k,\alpha}$.[1] For each $\bar{u}_k$, α_{k-1} is obtained by (3.11); and the probability of being in state α^* in 1975 is the sum of the probabilities of $\bar{u}$ ($P_{i,k=K,\alpha=0}$) over all $\bar{u}$'s which result (by 3.11) in $\alpha_{1975} = \alpha^*$.[2] In this way the probabilities of being in state α at 1975 are obtained for the entire range of the α's. These probabilities are denoted here as $P_{k,\alpha}$. The state probabilities $P_{k,\alpha}$ for 1980 and for any other stage (year) are obtained similarly by the following:

$$P_{k-5,\alpha^*} = \sum_{\alpha} P_{k,\alpha} \cdot \sum_{\bar{u} \in A_{k,\alpha,\alpha^*}} Q_k(\bar{u})$$

where A_{k,α,α^*} is the set of all $\bar{u}_k$ that, given α, results in $\alpha_{k-5} = \alpha^*$ by (3.11) and using the approximation explained in footnote 2 below.

The state probabilities depend strongly on the variance (σ^2) of the conditional probability distribution g (u | μ). Table 14 summarizes these state probabilities for the two variances used in the present computations. This table demonstrates that, as σ^2 is decreased, extreme values of α do not appear as the probability of reaching these extreme values is zero. Therefore, for the higher variance ($\sigma^2 = 20{,}000$), the relevant range of α is between ±150, while this range is reduced to ±50 for $\sigma^2 = 1{,}000$.

Note that the state probabilities are obtained on the basis of 1970 information, and they may change as time passes and additional data are obtained on population size. The importance of these probabilities to the decision–maker is that points on the strategy line for each project could be supplemented with probability statements. Using $\sigma^2 = 20{,}000$, the strategy line for the Coyote project (Figure 2) shows that this project should be initiated in 1980 if expected population is greater than or equal to 710,000. Since the initially projected population for 1980 is 560,000 (DF average), the critical α for the Coyote project to be undertaken in 1980 is 150,000. The probability of $\alpha \geqslant 150{,}000$ in 1980 is obtained from Table 14 in the row for 1980 and is equal to .00522 (= .00018 + .00504). Thus, using the optimal strategy viewed from 1970, the probability of starting the construction of the Coyote project in 1980 is approximately 1/2 of 1 percent. However, the probability of starting this project in 1985 is considerably greater—–about 28 percent—–when the critical α drops to 50,000. A similar probability

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[1] Recall from section III that $\bar{u}_k = \frac{1}{5} \sum_{i=k-4}^{k} u_i$.

[2] In the actual computation, α and $\bar{u}$ are assumed to take only a finite number ($\leqslant 15$) of values (grid points) within the range of ±300,000 which has been determined by the range of DF projections. Thus, for a given $\bar{u}_k$ and α_k, the resulting α_{k-5} obtained by (3.11) is approximated by the closest grid point. Therefore, for a given α_k, it is possible that more than one $\bar{u}_k$ will yield α^* at k − 5.

statement for the Warm Springs project for which the critical line starts after 2000 would be based on the steady–state probabilities.

TABLE 14

α State Probabilities $(P_{\alpha,k})^{a}$

α (1,000) / Year	0	50	100	150	200
$\sigma^2 = 20{,}000$					
1975	.59827	.17566	.02415	.00105	0
1980	.45696	.22000	.04268	.00504	.00018
1985	.43634	.22278	.05204	.00662	.00039
1990	.42000	.22460	.05695	.00786	.00061
.					
.					
.					
Steady state	.42000	.22700	.05824	.00815	.00061
$\sigma^2 = 1{,}000$					
Steady state[b]	.78860	.10470	0	0	0

[a] The state probabilities are presented for nonnegative α's only since $P_{\alpha,k} = P_{-\alpha,k}$.

[b] For $\sigma^2 = 1{,}000$, state probabilities converge to steady-state probabilities by 1975.

To conclude, the optimal strategy resulting from this stochastic model gives results similar to the deterministic plan in terms of order and timing of projects. However, the present sequential strategy calls for continuing observations on population growth in the service area as it materializes over time and gives decision rules for each time point in terms of the *minimal* expected population that justifies investment at that point. If population growth fails to reach that threshold, investment should be postponed. Thus, the order of projects is as in the deterministic plan, but the timing may differ depending on the rate of actual population growth.

APPENDIX

Adjustment of Parameters to 1970 Price Levels

The cost, price, and benefit parameters presented in section III were drawn from estimates developed with reference to different bases in time and thus different price levels. To use empirical parameters based on different price levels in our objective function (3.10) could bias staging results in favor of particular projects. Accordingly, we have adjusted all parameters entering (3.10) which are measured in monetary units to a common 1970 price base. This involves adjustments in project costs, flood and recreation annual benefits, and two components of the conservation water demand relation. The purpose of this Appendix is to present in summary form the adjusted parameter values used in our empirical application, specifying the inflation factors used in each case.

Project Costs

The unadjusted construction and annual cost estimates are given in Table 7 (section III). The U. S. Public Road Construction Cost Index was used for inflating construction costs to 1970 levels, and the U. S. Index of Associated Contractors of America was used for adjusting annual costs. The adjusted cost estimates are summarized in Appendix Table A–1. The empirical inflation factors applied are specified in the footnotes to Appendix Table A–1.

Flood Control and Recreation Benefits

Original annual flood and recreation benefit estimates appear in Table 8 (section III). Corresponding estimates adjusted to 1970 price levels are presented in Appendix Table A–2. The inflation factors for both classes of annual benefits are derived from the "all items" U. S. Consumer Price Index and are recorded in the source footnote for column 1 in Appendix Table A–2.

Conservation Water Demand

Industrial and Commercial Demand.––Since IC water demand is regarded as a water requirement independent of water price, no price inflation to 1970 is required. The empirical expression for the time path of requirements remains as in (3.17), repeated here for convenience:

$$Q_{it} = (9.185) \quad (1.015)^t. \tag{A.1}$$

Residential Demand.––Aggregate residential demand in terms of a 1960 price level is given by equation (3.6). The "housing" component of the U. S. Consumer Price Index is used to adjust (3.6) to a 1970 price level (U. S. Bureau of the Census, 1972, p. 348). The inflation factor is 1.31. Multiplying (3.6) through by 1.31, we obtain:

$$P_t = 520 + (164.7) \quad (1.021)^t - \frac{(4083.9) \quad (10^3)}{n_t} \quad Q_{rt}. \tag{A.2}$$

Agricultural Demand.––The data on payment capacity, which served as the basis for estimating the agricultural demand relation (3.8), are derived for a 1960–1964 base period (California Department of Water Resources, 1971b, p. 17). To adjust these payment capacities to a 1970 price level, a "fruit, truck crops, and other vegetables" component of the U. S. Index of Prices Received by Farmers has been used (Economic Statistics Bureau of Washington, D. C., 1974, pp. 140 and 141). This procedure yielded a multiplier of 1.1 where an average of this index for the five-year period 1960–1964 is taken as the base.

APPENDIX TABLE A-1

Yield and Cost Data Used in Computation
(Costs Adjusted to 1970 Cost Levels)

	Firm annual yield	Construction cost	Annual costs OMR
	1	2	3
	acre-feet	1,000 dollars	
Coyote	75,000	14,422	90
Warm Springs	115,000	94,034	790
Knights Valley			
I	50,000	67,000	442
I and II	150,000	131,750	2,021
I, II, and III	350,000	250,950	4,523

Sources:

Col. 1: Table 7, row 1. The figures for the Knights Valley project are rounded to the nearest 10,000.

Col. 2: Row 2, Table 7, entries multiplied by the following inflation factors based on the Public Road Construction Cost Index (U. S. Bureau of the Census, 1964, 1968, and 1972, pp. 743, 697, and 677, respectively). All construction costs in Table 7 except for Warm Springs were based on 1963 cost levels and are here multiplied by $I_{1970}/I_{1963} = 1.472$. The cost of Warm Springs was based on 1967 price levels, and its inflation factor is 1.264. For the first stage of the Knights Valley project, the cost of the conveyance system to Napa ($28.47 million) is deducted before multiplication by the index.

Col. 3: Row 3, Table 8, entries multiplied by inflation factors based on the Index of Associated General Contractors of America, which combines in a 40-60 ratio the costs of wages and materials (see *ibid.*). The corresponding multipliers for 1963 and 1967 are 1.454 and 1.256, respectively. Annual OMR costs of the conveyance system to Napa ($299,000) are deducted from the first stage of the Knights Valley project before multiplication by the inflation factor.

The appropriate adjustment is to multiply equation (3.8) through by 1.1. Doing so gives:

$$P_t = 82.5 - \bar{b}_t \; Q_{at} \tag{A.3}$$

where $\bar{b}_t = (1.1)\ b_t$. It was noted in discussing the empirical relation (3.8) that b_t varies from 1.5 in 1960 to .75 in 2020. After adjustment to express P_t in units approximating 1970 price levels, the slope coefficient $\bar{b}_t$ varies from 1.65 in 1960 to .825 in 2020.

Aggregate Demand.—Aggregating equations (A.1), (A.2), and (A.3) over quantity (Q) gives expression (3.9) adjusted to a common 1970 price level. The result is:

$$P_t \begin{cases} = \infty & \text{for } 0 \leqslant Q_t \leqslant (9.185)\ (1.015)^t \\[2ex] = 520 + (164.7)\ (1.021)^t + \dfrac{(37{,}511)\ (10^3)}{n_t}\ (1.015)^t - \dfrac{(4083.9)\ (10^3)}{n_t}\ Q_t, & \\ & \text{for } (9.185)\ (1.015)^t < Q_t \leqslant A_t \\[2ex] = \dfrac{[520 + (164.7)\ (1.021)^t]\ n_t\bar{b}_t + (37{,}511)\ (10^3)\ (1.015)^t\ \bar{b}_t + (336{,}922)\ (10^3)}{n_t\bar{b}_t + (4083.9)\ (10^3)} & \\ \quad - \dfrac{(4083.9)\ (10^3)\ \bar{b}_t}{n_t\bar{b}_t + (4083.9)\ (10^3)}\ Q_t & \\ & \text{for } A_t < Q_t \leqslant B_t \\[2ex] = 0, & \text{for } B_t < Q_t < \infty \end{cases} \tag{A.4}$$

where

$$A_t = \frac{[520 + (164.7)\ (1.021)^t]\ n_t + (37{,}511)\ (10^3)\ (1.015)^t - (82.5)\ n_t}{(4083.9)\ (10^3)}$$

and

$$B_t = \frac{[520 + (164.7)(1.021)^t]\, n_t\bar{b}_t + (37{,}511)(10^3)(1.015)^t\, \bar{b}_t}{(4083.9)(10^3)\,\bar{b}_t} + \frac{(336{,}922)(10^3)}{(4083.9)(10^3)\,\bar{b}_t}.$$

APPENDIX TABLE A-2

Annual Flood Control and Recreation Benefits
(Adjusted to 1970 Price Level)

Project	Flood control benefits	Recreation benefits
	1,000 dollars	
Coyote[a]	0	0
Warm Springs	2,326	1,686
Knights Valley	112	1,494

[a] No recreation or flood control benefits are attributed to Coyote since it consists of a second stage of an existing reservoir.

Source: Adjustment to 1970 price level involves multiplying the figures in row 1 of Table 9 by an inflation factor based on the "all items" U. S. Consumer Price Index (U. S. President, 1971, Appendix C, Table C-45, p. 249). For Warm Springs, the adjustment factor is $I_{1970}/I_{1967} = 1.163$; for Knights Valley, $I_{1970}/I_{1963} = 1.268$.

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