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Cattle Feedlot Marketing Decisions Under Uncertainty

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Uncertainty with respect to future prices, both for fat cattle and for feeder cattle, are important aspects in feedlot decision-making processes. This study illustrates, through the use of statistical decision theory, how such uncertainty can be considered directly in the decision process. Models are developed for determining decision rules for marketing cattle currently on feed or continuing their feeding for another time period, for the procurement of feeder animals, and for planning both procurement and marketing over a six-month planning horizon. A monthly price forecasting model serves as the basis for all the decision models. The predicting model is a recursive system involving forecasts of marketings of fat cattle, numbers of animals on feed, and other inventory levels to provide information for the primary price forecasting equations.

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CATTLE FEEDLOT MARKETING DECISIONS UNDER UNCERTAINTY[1,2]

INTRODUCTION

RISK AND UNCERTAINTY are dominant characteristics of many agricultural processing operations. Decisions on procurement, production, and marketing are made on the basis of imperfect knowledge about future conditions, and the stochastic nature of such conditions may result in a financial loss for the firm despite the care taken in making decisions. To the extent that risk and uncertainty can be incorporated into the decision-making process, the firm can reduce the chances of loss from unexpected market fluctuations.

This monograph's objectives are: 1) to illustrate through a practical application the use of probability theory in management decision making and 2) to develop a set of decision criteria to assist cattle feeders in making purchasing and marketing decisions when faced with uncertainty about future cattle prices.

THE PROBLEM

Uncertainty about future prices and the feedlot performance of cattle are challenges to cattle feeders faced with decisions on purchasing feeder cattle or marketing slaughter animals. Unexpected price changes, sickness, death loss, or simply low rates of gain of cattle on feed can lead to negative profits. The profitability of the cattle feeder's purchasing and marketing decisions hinges on his ability to anticipate future prices and to assess accurately the potential feedlot performance of the cattle on feed as well as his ability to operate efficiently the physical facilities of the feedlot.

The decision to place cattle on feed is based on a comparison of *expected* value of the cattle at the end of the feeding period with the current cost of feeder cattle plus expected feeding costs. The feedlot operator, therefore, must anticipate market conditions three to six months ahead and buy the age, quality, and type of feeder cattle that he expects will yield the most profit. Furthermore, he is faced with the possibility that his anticipations may be incorrect and expected positive returns may turn out to be negative.

Although operating practices vary among California feedlots, the "average" practice is to place 600-pound steers on feed with the intention of marketing them as 1,000-pound slaughter steers approximately 150 days later (Logan and King, 1966). In some instances,

[1] Submitted for publication September 20, 1971.

[2] This research project was developed jointly by the U. S. Department of Agriculture, Market Economics Division, Economics Research Service, and the University of California, Davis.

however, the cattle may be sold for slaughter at weights ranging from 800 pounds to more than 1,200 pounds. Thus, there is a range of about 400 pounds over which the operator must exercise the decision of whether to sell a particular lot of cattle at their current weight or to continue to feed the cattle at least another time period, which, for this study, is 30 days (one month). The feed-or-sell decision is based on a comparison of costs of feeding another 30 days with expected returns from the additional feeding. If the cattle are sold too soon, added profits may be foregone; however, if cattle are held on feed too long, profits can be decreased.

If the feeder can generate additional information concerning future conditions, he may be able to reduce the degree of random variation surrounding the possible outcomes of his decisions. Thus, limits may be placed on certain types of decisions as a result of such additional information. For example, feeder cattle may not be purchased unless predicted fat cattle prices six months ahead are at a pre-specified level, or have a certain probability of being attained.

To provide cattle feeders with information other than merely current price relationships, this study utilizes a monthly forecasting model for slaughter and feeder cattle prices developed previously (Bullock, 1968). The initial phase incorporates the information provided by the price forecasting model into a Bayesian decision framework to arrive at a set of marketing strategies. These strategies can be used by the cattle feeder to evaluate the feed-or-sell alternatives, given the current weight of a particular lot of cattle and the current price of slaughter cattle. The second phase utilizes the results of the price forecasting model to develop a set of decision criteria for purchasing feeder cattle. The final phase of the study combines the results of the previous two phases into a six-month planning model.

Although the decisions made in these planning models aim at increased profits, the various interrelationships among the decisions are not considered explicitly. For instance, the decision to retain cattle on feed rather than sell them for slaughter precludes the use of that pen space for new, lighter-weight feeder animals. However, it is possible that given feeder cattle and fat cattle price relationships, the optimum decision would be to replace older animals with younger ones whose weight gain will be greater than that of the older animals. These interrelationships are a separate study in themselves and are not considered here; thus, the model is of a partial nature.

CATTLE FEEDING OPERATIONS

Cattle feeding in California (see Logan and King, 1966, and Hopkin and Kramer, 1965) is characterized by large specialized feedlots. In 1969, 99 per cent of the fed cattle marketed in the State came from lots with capacities of 1,000 head or more. Moreover, about 82 per cent of the marketings originated in lots with capacities of 8,000 head or more. This is in sharp contrast to the major cattle feeding regions of the Midwest. For example, in 1969 in Iowa, lots with capacities of *less* than 1,000 head marketed 91 per cent of the fed cattle. Iowa lots with capacities exceeding 8,000 head accounted for less than 2 per cent of the marketings (USDA-SRS, 1970).

The large numbers of cattle fed per feedlot in California place added emphasis on the operator's purchasing and marketing decisions. Proper timing and better accuracy of these decisions can mean several thousand dollars in added revenue. Thus, in addition to achieving efficient gains for animals on feed, the feedlot operator must be aware of current and expected market conditions to purchase and sell cattle effectively.

Problems of price uncertainty

The primary source of risk in cattle feeding is imperfect knowledge about future prices. An experienced cattle feeder can estimate fairly accurately the cost of feeding a particular lot of cattle to the desired slaughter weight and the grades they will attain, and he can affect some of the factors of feedlot performance. However, future prices are dependent on many interrelated variables and beyond his control. Consequently, his information about future prices is less precise.

However, some knowledge of future slaughter cattle prices is necessary for decisions about placing cattle in the feedlot and for determining the best time to sell fed cattle. Information about future prices is probably most crucial for the purchasing decision because a three- to six-month forecast of slaughter cattle prices is needed.

The purchase decision is based on the *expected* feeding margin—the price per hundredweight received for the finished animal minus the price per hundredweight paid for the animal entering the feedlot. The break-even margin is defined as the margin necessary to cover all costs of feeding. The difference between realized margin and the break-even margin represents the profit (or loss) per hundredweight of fed steer. The accuracy of the feedlot operator's projection of slaughter cattle prices is critical. If future slaughter cattle prices are overestimated and additional cattle are fed, negative net returns may result. On the other hand, if prices are underestimated, positive returns may be foregone if cattle are not placed on feed.

Knowledge of future prices is also important in determining when to sell fed cattle. The decision to feed another month or sell at current weight is based on a comparison of the expected marginal revenue from continued feeding with the cost of feeding as indicated in equation (1).[3]

$$C \lesseqgtr P_{(t+1)}(W + G) - P_t W \qquad (1)$$

or

$$C \lesseqgtr P_t G + \Delta P (W + G)$$

where

C = cost of feeding another month,
W = current weight,
G = weight gain,
P_t = current slaughter price,
$P_{(t+1)}$ = price next month,
$\Delta P = P_{(t+1)} - P_t$

A particular lot is continued on feed only if $C < P_t G + \Delta P (W + G)$. As in the purchase decision, the primary source of risk in the selling decision is the price change that may occur.

Some of the uncertainty about future price movements can be removed by forward contracting for purchase and sale of cattle[4] and by hedging operations in the futures market. However, it is difficult for California cattle feeders to hedge effectively in the futures mar-

[3] This is a simplified version of the marketing decision problem because replacement aspects are ignored. For a discussion of replacement decisions, see Faris (1960).

[4] Logan and King (1966, pp. 21–23) reported that about half of the feedlots surveyed used contracts for purchasing feeder cattle and advance contracts (30 days) for sale of fed cattle were used for 73 per cent of cattle marketed.

ket because a viable West Coast futures market does not exist and hedging operations must be transacted in the Midwestern market. Futures contracts have rigid specifications as to weight, grade, and location of cattle that can be delivered under contract. Thus, to utilize the Midwestern futures market, the California cattle feeder has to adjust Midwestern cattle prices for locational differences and for the quality of cattle in his feedlot. Although slaughter cattle prices in California and the Midwest are interrelated, they are not perfectly correlated. Thus, in the short run, prices in one market may be declining while in the other market prices may be holding steady or even increasing slightly.[5] In such cases, price movements adverse to the California cattle feeder are magnified if he is using the Midwestern market for hedging operations. Consequently, while the futures market may reduce risk, it does not completely remove price uncertainty for the cattle feeder.

Other sources of risk

Poor feedlot performance is another important source of risk for the cattle feeder. Scientific management practices may have helped to reduce sickness and death loss of cattle on feed. Veterinarians and nutrition experts frequently are employed by large feedlots to reduce these risks, but they have not been eliminated.

Typically, the cattle feeder operates on a narrow margin of profit, basing his purchase decision on what he thinks the cost per pound of gain will be for the feeder cattle and their expected value at the end of the feeding period. If the feeder cattle do not gain as efficiently as he had anticipated, or if feed prices rise unexpectedly, added cost per pound of gain may eliminate expected profits, regardless of the accuracy of his price expectations. Similarly, if the cattle do not reach the planned slaughter grades, their value at the end of the feeding period will be less than expected and negative profits may result.

THE DECISION MODEL

The problem of decision making under uncertainty can be characterized as a decision maker faced with choosing the optimal course of action, A_i, from a set of m possible actions. The outcomes of these various actions are dependent on the occurrence of alternative states of nature Θ_j, $j = 1, 2, \ldots, n$. The states of nature are values of an exogenous factor that directly affects the outcome of a particular action but is beyond the control of the decision maker; at least, this factor cannot be controlled with certainty. For example, if the set of actions represents different rates of fertilizer applications for corn, the states of nature might be alternative levels of rainfall. Thus, for each possible action A_1, $A_2, \ldots, A_m$, there are n potential outcomes, one for each state of nature. Each outcome, λ_{ij}, can be represented as a point in an action-state plane, $\lambda_{ij} = (A_i, \Theta_j)$. The matrix formulation of the outcome plane is presented in table 1.

For example, the outcome (profits) of a decision to feed two types of steers (low quality and high quality) will depend on the prices of slaughter cattle at the end of the feeding period. Thus, Θ_1

[5] Divergent movements are limited by the amount of transportation costs between the two markets because intermarket shipments become profitable if prices differ by more than transfer costs. However, price movements within this range could exceed feeding margins in some cases.

TABLE 1
MATRIX REPRESENTATION OF OUTCOME PLANE

Action	States of nature									
	Θ_1	Θ_2	•	•	•	Θ_j	•	•	•	Θ_n
A_1	λ_{11}	λ_{12}	•	•	•	λ_{1j}	•	•	•	λ_{1n}
A_2	λ_{21}	λ_{22}	•	•	•	λ_{2j}	•	•	•	λ_{2n}
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
A_i	λ_{i1}	λ_{i2}	•	•	•	λ_{ij}	•	•	•	λ_{in}
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
A_m	λ_{m1}	λ_{m2}	•	•	•	λ_{mj}	•	•	•	λ_{mn}

may represent high slaughter-cattle prices; Θ_2, average prices; and Θ_3, low prices. The outcome of decisions A_1 (feed high-quality steers) and A_2 (feed low-quality steers) will depend on which value of Θ occurs (cost per pound of gain is assumed to be known with certainty in both cases). This decision problem is then as follows:

Action	*States of nature*		
	Θ_1 *(high prices)*	Θ_2 *(average prices)*	Θ_3 *(low prices)*
A_1 (feed high-quality steers)	λ_{11}	λ_{12}	λ_{13}
A_2 (feed low-quality steers)	λ_{21}	λ_{22}	λ_{23}

where λ_{12} is the profit per head from feeding high-quality steers when average prices are received at the end of the feeding period.

To make rational and consistent decisions about the action-state-outcome combinations, a utility index or some sort of preference ordering must be assigned to the set of outcomes. If the decision maker's preferences among the outcomes are consistent with von Neumann-Morgenstern utility axioms (see also Luce and Raiffa, 1965, pp. 23–31) it is possible to define a utility function, $u_{ij} = u(\lambda_{ij})$, that will map the outcomes into a utility plane.

Von Neumann and Morgenstern (1947) show that if:

1. the individual has a complete and transitive preference ordering over the set of all possible prospects, that is, (a) for any two prospects u and v, one and only one of the following relations holds:

$$u = v,\ u > v,\ u < v$$[6]

   (b) $u > v$, $v > w$ implies $u > w$
2. $u < w < v$ implies the existence of an $\alpha(u) + (1 - \alpha)v < w$, and $u > w > v$ implies the existence of an $\alpha(u) + (1 - \alpha)v > w$, where

[6] Where: = implies indifference between prospects,
> is read as "is preferred to," and
< is read as "is not preferred to."

TABLE 2
MATRIX FORMULATION OF DECISION PROBLEM UNDER UNCERTAINTY

Action	States of nature									
	Θ_1	Θ_2	•	•	•	Θ_j	•	•	•	Θ_n
A_1	u_{11}	u_{12}	•	•	•	u_{1j}	•	•	•	u_{1n}
A_2	u_{21}	u_{22}	•	•	•	u_{2j}	•	•	•	u_{2n}
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
A_i	u_{i1}	u_{i2}	•	•	•	u_{ij}	•	•	•	u_{in}
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
A_m	u_{m1}	u_{m2}	•	•	•	u_{mj}	•	•	•	u_{mn}

$0 < \alpha < 1$, and

3. if it is irrelevant whether a combination of two prospects is obtained in two successive steps—first the probabilities α, $1 - \alpha$, then the probabilities β, $1 - \beta$; or in one operation with the probabilities γ, $1 - \gamma$ where $\gamma = \alpha\beta$ (that is, complex choices can be partitioned into simpler choices to facilitate evaluating preferences)

$$\alpha u + (1 - \alpha)v = (1 - \alpha)v + \alpha u$$

and

$$\alpha[\beta u + (1 - \beta)v] + (1 - \alpha)v = \gamma u + (1 - \gamma)v$$

then there exists a utility function u on the set of prospects.

In other words, for each prospect P_i there exists a number $u_i = u(P_i)$ which is called the utility of P_i. This function has the following properties (Chernoff and Moses, 1959):

(a) $u(v) > u(w)$ if and only if the individual prefers v to w.

(b) If P_k is a prospect of receiving v with probability α or w with probability $(1 - \alpha)$ then $u(P_k) = \alpha u(v) + (1 - \alpha)\, u(w)$.

As a matter of practical application, it is usually assumed that the utility function is linear with respect to money over the relevant range. Consequently, maximization of monetary gain is equivalent to maximizing utility.

Thus, the decision problem can be seen as stated in table 2. Given a set of possible actions, A, the set of alternative states of nature, Θ, and the utility index u_{ij}, associated with the selection of action A_i and the occurrence of Θ_j (outcome λ_{ij}), select the action that is in some sense optimal—where optimality is defined by the particular decision criterion used. Various decision criteria are available, many of which deal with decisions with no knowledge at all about the states of nature.

However, most of these decision criteria have serious shortcomings as discussed by Luce and Raiffa (1965, pp. 278–286). See also Chernoff (1954) and Radner and Marschak (1954).

Bayesian decision theory

Few decision problems fall into the category of complete uncertainty, i.e., where the decision maker has no knowledge of the likelihood or distribution of Θ. Given the volume of public and private information currently available, some *a priori* information regarding the relative frequency of Θ in the past generally can be obtained. Thus, emphasis in decision theory has shifted to the estimation of Bayesian strategies;[7] i.e., the selection of optimal actions based on some *a priori* information, either objective or subjective, about the probability distribution of the states of nature, $P(\Theta)$.

The Bayesian approach to decision making can be stated as follows: Given a set of m possible actions, the set of n alternative states of nature, and the utility index associated with each outcome, along with a vector of *a priori* information about the relative frequency of Θ,

$$P(\Theta) = \begin{matrix} P(\Theta_1) \\ P(\Theta_2) \\ \vdots \\ P(\Theta_j) \\ \vdots \\ P(\Theta_n) \end{matrix} \quad \text{where } P(\Theta_j) \text{ is the } a \text{ } priori \text{ probability that state } \Theta_j \text{ will occur}$$

select the action A_i for which *expected* utility $\hat{u}_i = \sum_j u_{ij}P(\Theta_j)$ is a maximum.

The *a priori* information can be any information that the decision maker has about the relative frequency of Θ. This information is expressed in the form of a probability distribution $P(\Theta)$ that provides some indication of the likelihood of a particular value of Θ (state of nature) occurring. It may be nothing more than a subjective evaluation of the probabilities by the decision maker, or it may be derived mathematically from data on the relative frequency of Θ in the past.

In addition to the *a priori* knowledge of the probability distribution, $P(\Theta)$, it may be possible for the decision maker to gain additional information about the likelihood of a particular state Θ_j by performing an experiment Z (with results Z_k, $k = 1, 2, \ldots, n$) that serves as a predictor of Θ.[8] That is, it may be possible to construct a conditional probability distribution, $P(\Theta|Z)$, which incorporates the *a priori* information, $P(\Theta)$, with information about the past performance of Z as a predictor of Θ. The *a posteriori* probability distribution, $P(\Theta|Z)$, can be calculated using Bayes' Formula:[9]

$$P(\Theta|Z) = \frac{P(Z|\Theta)(P\Theta)}{P(Z)}$$

The experimental information expands our knowledge about the likelihood of Θ from the $P(\Theta)$ vector to an (nxn) matrix of conditional probabilities (table 3), where $P(\Theta_j|Z_k)$ is the probability of Θ_j occurring given Z_k as the experimental result (prediction of Θ). If the experiment Z is a perfect predictor of Θ, table 3 will consist of ones along the diagonal and zeros elsewhere.

With data provided by the experiment, the Bayesian strategy becomes: Given a projection of Θ (for example, Z_k) select the action A_i for which the expected utility

$$\hat{u}_i^k = \sum_j u_{ij}P(\Theta_j|Z_k) \qquad (3)$$

is a maximum. Thus, the Bayesian

[7] (Jeffery, 1965); (Raiffa and Schlaifer, 1961); (Weiss, 1961); (Luce and Raiffa, 1965) and (Chernoff and Moses, 1959).

[8] The experiment, Z, can be anything that is used as an estimator of Θ. It may consist of simply observing the current state of nature Θ_j and assuming that the value of Θ at the time of payoff will also be Θ_j. The price forecasting model developed in the following section functions as the experiment for this study.

[9] For a derivation of Bayes' Formula, see Hoel (1962, p. 16). This procedure is used to calculate *a posteriori* probability distributions in this study. For other applications see Eidman *et al.* (1968) and Dean *et al.* (1966). Depending on the nature of the experimental data, it may also be possible to estimate $P(\Theta|Z)$ directly without the use of Bayes' Formula.

TABLE 3

MATRIX OF *A POSTERIORI* INFORMATION

States	Experimental results									
	Z_1	Z_2	•	•	•	Z_k	•	•	•	Z_n
Θ_1	$P(\Theta_1\|Z_1)$	$P(\Theta_1\|Z_2)$	•	•	•	$P(\Theta_1\|Z_k)$	•	•	•	$P(\Theta_1\|Z_n)$
Θ_2	$P(\Theta_2\|Z_1)$	$P(\Theta_2\|Z_2)$	•	•	•	$P(\Theta_2\|Z_k)$	•	•	•	$P(\Theta_2\|Z_n)$
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
Θ_j	$P(\Theta_j\|Z_1)$	$P(\Theta_j\|Z_2)$	•	•	•	$P(\Theta_j\|Z_k)$	•	•	•	$P(\Theta_j\|Z_n)$
•	•	•	•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•	•	•	•
Θ_n	$P(\Theta_n\|Z_1)$	$P(\Theta_n\|Z_2)$	•	•	•	$P(\Theta_n\|Z_k)$	•	•	•	$P(\Theta_n\|Z_n)$

strategy consists of a set of optimal actions, at least one for each experimental result.[10]

Value of the data

The derivation of Bayesian decisions by using only the *a priori* probability distribution $P(\Theta)$ is referred to as the "no data" problem. Decision problems using *a posteriori* distributions are called "data" problems. The difference in expected incomes resulting from using the "data" strategy bundle relative to the "no data" strategy can be interpreted as the value of the data, i.e., the value of the information provided by the experiment.

The expected value of the "no data" strategy is defined above as $\hat{u}_i = \sum_j u_{ij}P(\Theta_j)$. The expected value of following the "data" strategy is calculated by multiplying the expected value of the optimum action for each experimental result by the probability of observing the appropriate experimental result, $P(Z)$, and summing over all possible results

$$\sum_k [\sum_j u_{ij}P(\Theta_j|Z_k)]P(Z_k) \qquad (4)$$

The expression in brackets was defined in equation (3) as $\hat{u}_i^k$ (expected utility of action A_i given Z_k as a prediction of Θ). Thus, the above expression reduces to $\sum_k \hat{u}_i^k P(Z_k)$. Therefore, the value of the data is defined as

$$V = \sum_k [\sum_j u_{ij}P(\Theta_j|Z_k)]P(Z_k) - \sum_j u_{ij}P(\Theta_j)$$

$$V = \sum_k \hat{u}_i^k P(Z_k) - \hat{u}_i.$$

MODEL FORMULATION

Within this general framework of decision theory, four models are set up as a framework for analysis. Models I and II are short-run models and deal only with marketing decisions; Model III involves longer-run purchasing decisions; and Model IV combines marketing and purchasing decisions for a six-month planning horizon.

Model I is a direct application of Bayesian decision theory to the problem of feedlot *marketing* decisions. It is designed to determine the minimum *expected* price change required to induce feeding a particular lot of cattle another month, given the current weight of the cattle and current slaughter cattle prices. The model incorporates information

[10] It is possible that two or more actions could have the same expected utility for a given experimental result.

about the cost of the additional gain and expected slaughter grade of the cattle 30 days hence with *a posteriori* information (in the form of probability distributions) about the accuracy of the price forecasting model to arrive at a set of feed-or-sell decision rules.

Model II is an extension of Model I. For animals weighing less than 1,000 pounds, it is not unreasonable to consider extending the feeding period another 60 days. Furthermore, it is conceivable that a sell decision could be generated by Model I when a one-month price projection is considered but that it might be profitable to continue feeding the animals if we consider expected prices 60 days hence. Model II, therefore, is constructed to evaluate the feed-or-sell decision based on 60-day price projections. This model is applicable only if (a) current weight of the cattle is less than 1,000 pounds and (b) a sell decision arises in Model I.

Model III develops a set of buy-or-not-buy decision criteria for feeder cattle based on expected feeding margins. Estimates of cost per pound of gain and proportion of cattle feeding to choice grade are combined with projected slaughter cattle prices to determine expected feeding margins.

Model IV, a six-month planning model, incorporates the decision rules developed in the first three models into a simulation model. Model IV simulates the buying, feeding, and selling activities six months into the future, given the capacity of the feedlot, current inventories of cattle on feed by weights, and projected feeder and slaughter cattle prices. This information should be helpful to the feedlot operator in making forward arrangements for financing, feed acquisition, and contracting for purchase of feeder cattle and/or sale of slaughter cattle.

DATA REQUIREMENTS

Model IV requires the same data as the first three models plus longer-run projections of prices; therefore, a discussion of data needs for this model automatically covers the needs of the first three models. To make tentative decisions about purchases and sales six months in advance, feeder cattle prices must be projected six months into the future and slaughter cattle prices 11 months ahead. For example, a tentative decision regarding placements six months ahead requires a six-month projection of feeder cattle prices plus an estimate of slaughter cattle prices five months later, at the end of the proposed feeding period (i.e., 11 months in advance of the planning date).

Two additional sets of information are required to develop strategies for the marketing and purchase decisions: (a) cost per pound of gain as the weight of the animal increases and (b) the proportion of fed cattle that can be expected to grade Choice or better at alternative slaughter weights. Aside from price changes, these are the primary variables in the marketing and purchase decisions.

Cost per pound of gain increases as weight of the animal increases because a larger proportion of feed intake is required just for maintenance at greater weights (National Academy of Sciences, NRC, 1963; Garrett *et al.*, 1959). Almost twice as much feed is required per pound of gain for 1,200-pound steers as for 600-pound steers. Thus, in some instances, feeding to heavier weights may not be feasible because the cost per pound of gain may exceed slaughter cattle prices.

This rising cost per pound of gain, however, may be offset to some extent as additional Good grade steers attain Choice grade, because the proportion of slaughter steers grading Choice increases (and thus their value increases), *ceteris paribus*, as weight increases.

The input requirements, then, needed to develop the models formulated above can be summarized as follows:

1. A monthly price forecasting model to project
   (a) slaughter cattle prices 11 months ahead and
   (b) feeder cattle prices six months ahead.
2. *A posteriori* probability distribution of price changes, given projections of the price forecasting model.
3. Data relating the cost per pound of gain to weight of steer.
4. Data relating proportion of cattle grading Choice to slaughter weight.

In addition, a probability distribution of price changes in the past will be used as the basis for a "no data" strategy with which to compare the results of "data" strategy utilizing the price forecasting model.

APPLICABILITY AND GENERAL SPECIFICATIONS

The decision rules developed in this study are based on typical cost and production relationships of California feedlots. Because not all California feedlots have the same cost structure or follow the same operating procedures, the question arises how applicable decision rules based on average relationships are to specific problems faced by an individual feedlot operator.

The applicability of the decision rules to a wide range of decision problems depends on how sensitive the models (used to derive the rules) are to the above mentioned variables. Do slight changes in cost relationships or variations in feed prices give rise to a different set of decision rules? A sensitivity analysis of the models (explained in detail later) indicates that the same decision rules would be derived for a range of feeding costs. Thus, the rules developed from "typical" or average cost and production relationships should have a rather general application to decision problems faced by California feedlot operators.

The "typical" feedlot is assumed to purchase 600-pound Good grade feeder steers at prices reflected by the Stockton, California, market and to sell Good and Choice grade slaughter steers at prices indicated by the El Centro, California, market. El Centro and Stockton were selected as the representative markets for this study because they are important markets in the State and time series of price data are available for use in developing the price forecasting model. Their selection is not a limiting factor, however, because prices of feeder cattle and slaughter cattle throughout the State are affected by the same factors. Prices between geographic points in the State are interdependent and price changes from one time period to the next (the basis for this study) will be essentially the same for each point.

PRICE FORECASTING MODEL

A brief summary of the monthly price forecasting model, developed in detail elsewhere, is given here. However, the equations used in this application have been revised with more recent data than those used in Bullock (1968). Essentially, the model is recursive in nature with prices being related to predicted values of other variables. The model predicts monthly prices of 900- to 1,100-pound Choice grade slaughter steers at El Centro as a function of lagged prices and predicted marketings of fed cattle in California, Arizona, Texas, Colorado, and the North Central region, based on the period 1960–69. Marketings, in turn, depend on the number of cattle and calves on feed at the beginning of a particular quarter. The latter variable requires prediction, also, when prices are to be forecast further ahead than the current quarter.

The basic structure of the price forecasting model is outlined by the following equations:

Price Forecasting Equation ($\hat{P}_{ji}$)

$$\hat{P}_{ji} = f_{ji}(\hat{M}_{jk}, \hat{P}_{(ji)-1}, P_{(ji)-12}, Q_1, Q_2, Q_3) \quad (6)$$

(Choice grade slaughter cattle prices are predicted as a function of projected marketings of fed cattle in various regions, lagged prices of Choice grade steers, and quarterly dummy variables.)

Fed Cattle Marketings ($\hat{M}_{jk}$)

$$\hat{M}_{jk} = g_{jk}(\hat{W}_{nhk}, Q_1, Q_2, Q_3, T) \quad (7)$$

(Fed cattle marketings in region k are projected as a function of predicted or actual cattle on feed by weight group in the region plus quarterly variables and a linear time trend.)

Cattle on Feed Projections ($\hat{W}^{\alpha}_{nhk}$)

$$\hat{W}^{\alpha}_{nhk} = f_k(S_k, C_k, W_{1k}, W_{2k}, W_{3k}) \quad (8)$$

(A projection n quarters ahead of cattle on feed in weight group h for region k is a function of January 1 inventories of steers and calves and cattle on feed by weight group (excluding $\geq$ 1,100 pounds) in region k.)

Steer Inventory ($\hat{S}_{(t+1)k}$)

$$\hat{S}_{(t+1)k} = g_k(C_{tk}, BC_{(t-1)k}, M) \quad (9)$$

(Steer inventory on January 1 for the coming year is a function of January 1 inventory of calves for the current year in the region, the January 1 inventory of beef cows for the previous year, and the average Kansas City-Chicago feeding margin for the current year up to the time of projection.)

Calf Inventory ($\hat{C}_{(t+1)k}$)

$$\hat{C}_{(t+1)k} = h_k(\overline{PP}'_{(t-1)}, BC_{tk}, BH_{tk}, \overline{PP}') \quad (10)$$

(Calf inventory on January 1 for the coming year is a function of the average price of feeder steers at Kansas City the preceding year, inventories of beef cows and beef heifers on January 1 of the current year in region k, and the average price of feeder steers at Kansas City for the current year up to the time of projection.)

The symbols used in the above functions are as follows:

BC_{tk} = January 1 inventory of beef cows for current year in region k (1,000 head).

$BC_{(t-1)k}$ = January 1 inventory of beef cows for previous year in region k (1,000 head).

C_{tk} = January 1 inventory of calves for current year in region k (1,000 head).

$\hat{C}_{(t+1)k}$ = projection of January 1 inventory of calves less than one year old for the coming year in region k (1,000 head).

h = 1, 2, 3, 4, where
1 = 500 to 699 pounds

2 = 700 to 899 pounds
3 = 900 to 1,099 pounds
4 = more than 1,100 pounds.

i = the length (in months) of the projection being made.

j = the month of the quarter and equals 1, 2, or 3.

k = the feeding region.

n = the length (in quarters) of projections of cattle on feed.

M = average Kansas City-Chicago feeding margin for the current year up to the time of projection (dollars per hundredweight).

$\hat{M}_{jk}$ = projected marketings of fed cattle for the j^{th} month in region k (1,000 head).

$\overline{PP'}$ = average price of feeder steers (all weights and grades) at Kansas City for current year up to time of projection (dollars per hundredweight).

$\overline{PP'}_{(t-1)}$ = average price of feeder steers (all weights and grades) at Kansas City in preceding year (dollars per hundredweight).

$\hat{P}_{ji}$ = projected price for the j^{th} month i months ahead (dollars per hundredweight).

$\hat{P}_{(ji)-1}$ = projected price for month preceding the (ji) projection. If $i = 1$, i.e., a one month projection, the actual lagged price is used (dollars per hundredweight).

$P_{(ji)-12}$ = price 12 months previous to the month for which price is being projected (dollars per hundredweight).

Q_1, Q_2, Q_3 = quarterly dummy variables.

S_k = January 1 inventory of steers (1,000 head) one year old and older in region k or its major supply region.

$\hat{S}_{(t+1)k}$ = projection of January 1 inventory of steers for the coming year in region k (1,000 head).

t = current year.

T = a linear time trend = 0 for the first quarter of 1960 and increases by 1 each quarter. $T = 36$ for the first quarter of 1969.

W_{hk} = cattle on feed in weight group h ($h = 1, 2, 3, 4$) in region k at the beginning of either the current quarter or total for most recent two quarters (excluding $h = 4$).

$\hat{W}^{\alpha}_{nhk}$ = an n-quarter projection of cattle on feed (1,000 head) in weight group h in region k. α refers to the quarter from which the projection is made and equals 1, 2, 3, 4.

The model is subdivided by quarters because cattle and calves on feed data are available only by quarters. Marketings are forecast with separate equations for the first, second, and third months of the quarter. Prices, in turn, are also forecast with a separate equation for each month of a quarter and with a separate equation depending on the number of months ahead for which the prediction is being made. In this sense, there is one equation for a four-month projection of prices in the first month of a quarter, another equation for a four-month projection of the second month of the quarter, and so forth. The coefficients of price predicting equations are given in table 4.

The use of separate equations for each type of forecast precludes the use of the standard error of the estimate in deriving the needed probability distributions for the decision problem, as will be discussed later. However, the standard errors of the estimate are given in table 4.

TABLE 4

PRICE FORECASTING EQUATIONS FOR CHOICE GRADE, 900–1,100 POUND SLAUGHTER STEERS, EL CENTRO, CALIFORNIA

Equation	Length of price projection	Month of quarter	Constant term	Lagged prices		Projected marketings of fed cattle					Quarterly dummy variables*			R^2	$S\hat{y}$	$\bar{Y}$
				$\hat{P}_{(t-1)}$	$P_{(t-12)}$	California	Arizona	Texas	Colorado	North central	Q_1	Q_2	Q_3			
			a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}			
	months														*dollars/cwt*	
1.	1	1	11.5938	.7115	.0846	−.0122	−.0253	−.0069	.0606	−.0075	.9529	−.4535	.1666	.9206	.6692	25.59
				(.0669)†	(.0886)	(.0134)	(.0496)	(.0099)	(.0245)	(.0040)	(.6426)	(.7124)	(.4935)			
2.	1	2	28.3505	.8818	−.0900	−.0147	−.0807	.0053	.1051	−.0260	−.0338	−.6440	.2656	.9421	.6477	25.61
				(.0837)	(.0786)	(.0115)	(.0376)	(.0124)	(.0299)	(.0063)	(.3772)	(.7741)	(.2105)			
3.	1	3	10.2923	.8583	−.1552	−.0237	.0070	−.0105	.0611	−.0034	−.3786	−.5616	.1421	.9182	.7860	25.83
				(.0908)	(.0917)	(.0136)	(.0440)	(.0133)	(.0377)	(.0049)	(.3542)	(.6934)	(.4218)			
4.	2	1	− 2.5770	.6490	−.1199	.0217	.0198	.0303	−.0995	.0148	.4833	−.0155	.5964	.8307	.9775	25.59
				(.1162)	(.1705)	(.0213)	(.0654)	(.0164)	(.0406)	(.0136)	(1.6437)	(1.4582)	(.9633)			
5.	2	2	34.1724	.7553	−.0375	−.0201	−.0776	.0001	.1364	−.0313	.1128	−1.1682	.4036	.8500	1.0587	25.42
				(.1620)	(.1310)	(.0193)	(.0634)	(.0217)	(.0500)	(.0109)	(.6392)	(1.3083)	(.3481)			
6.	2	3	11.7481	.8143	−.1735	−.0258	.0077	−.0116	.0655	−.0032	−.4066	−.6332	−.0876	.8314	1.1287	25.83
				(.1527)	(.1363)	(.0197)	(.0640)	(.0193)	(.0558)	(.0075)	(.5117)	(1.0118)	(.6075)			
7.	3	1	1.1316	.7104	−.1935	.0231	−.0315	.0394	−.1056	.0136	−.0229	.4027	.9101	.8097	1.0362	25.59
				(.1430)	(.1815)	(.0226)	(.0742)	(.0170)	(.0430)	(.0144)	(1.7421)	(1.5612)	(.0161)			
8.	3	2	20.6716	1.3981	−.0631	−.0867	−.2037	−.0312	.1513	−.0148	−2.2121	−.2640	.9010	.7552	1.3529	25.42
				(.2469)	(.1652)	(.0273)	(.1372)	(.0259)	(.0538)	(.0152)	(2.4676)	(2.6874)	(.7407)			
9.	3	3	5.3696	.9033	−.2009	−.0267	.0439	−.0105	.0182	.0038	−.5194	−1.1191	.0763	.7790	1.3116	25.80
				(.2533)	(.1687)	(.0234)	(.0792)	(.0226)	(.0772)	(.0110)	(.6184)	(1.2498)	(.7084)			
10.	4	1	5.5081	.7537	−.1233	.0321	−.0410	.0356	−.0887	.0048	−.9308	1.2946	.3601	.8020	1.0650	25.54
				(.1668)	(.1876)	(.0234)	(.0793)	(.0180)	(.0458)	(.0150)	(1.8119)	(1.6637)	(.0430)			
11.	4	2	18.3051	1.4369	−.0725	−.0763	−.2197	−.0284	.1448	−.0138	−2.2030	.1270	.7757	.7575	1.3464	25.42
				(.2515)	(.1647)	(.0267)	(.1377)	(.0256)	(.0528)	(.0151)	(2.4558)	(2.6708)	(.7408)			
12.	4	3	5.8491	.6214	−.1875	−.0267	.1103	.0110	−.0205	.0077	−.3166	−1.2486	.2793	.6795	1.5795	25.80
				(.2469)	(.1973)	(.0348)	(.1318)	(.0272)	(.0454)	(.0242)	(2.6940)	(2.9959)	(1.2102)			
13.	5	1	11.8464	.5816	.0433	−.0225	.1249	−.0038	.0129	−.0054	2.4346	−1.0040	.6820	.7129	1.2826	25.54
				(.2564)	(.1758)	(.0322)	(.0957)	(.0157)	(.0461)	(.0100)	(2.0212)	(1.7834)	(1.0087)			
14.	5	2	14.7115	1.4145	−.0592	−.0585	−.1690	−.0292	.1243	−.0136	−2.0167	.3891	.5451	.7504	1.3789	25.37
				(.2640)	(.1691)	(.0274)	(.1409)	(.0266)	(.0519)	(.0156)	(2.5760)	(2.7994)	(.7733)			
15.	5	3	7.6951	.6665	−.1902	−.0245	.0962	.0101	−.0144	.0048	−.5094	−.9439	.2642	.6910	1.5509	25.80
				(.2416)	(.1930)	(.0342)	(.1300)	(.0267)	(.0449)	(.0234)	(2.5859)	(2.9144)	(1.1677)			
16.	6	1	11.8860	.6012	.0318	−.0245	.1253	−.0037	.0130	−.0054	2.5070	−1.0917	−.7287	.7237	1.2582	25.54
				(.2375)	(.1724)	(.0312)	(.0913)	(.0154)	(.0448)	(.0096)	(1.9188)	(1.7429)	(.9704)			
17.	6	2	− 1.2277	1.1974	−.1460	−.0035	−.1514	.0125	−.0021	−.0067	−1.4458	1.6460	.0545	.6639	1.6000	25.37
				(.3702)	(.2123)	(.0334)	(.1729)	(.0264)	(.0622)	(.0157)	(1.9524)	(2.3319)	(.6917)			

* For a one-month projection actual lagged price is used.

† The number in parentheses is the standard error of the coefficient.

TABLE 4 (Continued)

PRICE FORECASTING EQUATIONS FOR CHOICE GRADE, 900–1,100 POUND SLAUGHTER STEERS, EL CENTRAO, CALIFORNIA

Equation	Length of price projection	Month of quarter	Constant term	Lagged prices		Projected marketings of fed cattle					Quarterly dummy variables*			R^2	$S\hat{y}$	$\bar{Y}$
				$\hat{P}_{(t-1)}$	$P_{(t-12)}$	California	Arizona	Texas	Colorado	North central	Q_1	Q_2	Q_3			
			a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}			
	months														*dollars/cwt*	
18	6	3	12.4075	.7185	−.1866	−.0178	.0690	.0137	−.0136	−.0010	−.9467	−.0768	.2178	.6965	1.5634	25.78
				(.2633)	(.1945)	(.0352)	(.1358)	(.0267)	(.0454)	(.0233)	(2.5490)	(2.9064)	(1.1805)			
19	7	1	11.1214	.6434	.0076	−.0281	.1203	−.0032	.0094	−.0040	2.3738	−1.3264	−.6967	.7364	1.2518	25.53
				(.2438)	(.1720)	(.0306)	(.0903)	(.0154)	(.0452)	(.0098)	(1.9174)	(1.7402)	(.9734)			
20	7	2	.6108	1.1549	−.1503	−.0031	−.1257	.0129	−.0016	.0046	−1.1495	1.3704	.0186	.6995	1.5868	25.37
				(.3468)	(.2104)	(.0331)	(.1671)	(.0260)	(.0616)	(.0154)	(1.9073)	(2.2856)	(.6879)			
21	7	3	21.9801	.5564	−.1904	−.0024	−.0138	.0278	.0648	−.0127	−.9683	−1.7788	−.1159	.6557	1.6653	25.78
				(.3493)	(.2231)	(.0532)	(.1792)	(.0254)	(.0606)	(.0086)	(1.3834)	(2.2317)	(1.5087)			
22	8	1	−9.6141	1.1474	.1396	−.0045	.0429	−.0296	.0395	.0001	1.5791	−.3272	−.6156	.7069	1.3198	25.53
				(.2877)	(.1832)	(.0280)	(.9053)	(.0244)	(.0478)	(.0035)	(1.2263)	(1.4530)	(.9252)			
23	8	2	.4934	1.2034	−.1510	−.0008	−.1519	.0122	.0054	.0040	−1.2143	1.5881	.0317	.6780	1.5951	25.38
				(.3604)	(.2120)	(.0334)	(.1758)	(.0261)	(.0625)	(.0156)	(1.9843)	(2.3638)	(.7070)			
24	8	3	22.1637	.5452	−.1926	−.0024	−.0114	.0284	.0630	−.0125	−.9845	−1.7488	−.0995	.6559	1.6648	25.78
				(.3410)	(.2226)	(.0531)	(.1785)	(.0252)	(.0605)	(.0086)	(1.3762)	(2.2343)	(1.5024)			
25	9	1	−9.2611	1.1371	.1356	−.0079	.0525	−.0308	.0418	.0001	1.7292	−.5222	−.7025	.7034	1.3278	25.53
				(.2897)	(.1842)	(.0280)	(.0965)	(.0248)	(.0483)	(.0036)	(1.2461)	(1.4602)	(.9390)			
26	9	2	16.1307	.9483	−.2233	−.0278	−.1300	.0102	.0601	−.0044	−.8380	.0403	1.1911	.6756	1.6009	25.38
				(.3338)	(.1841)	(.0365)	(.1127)	(.0262)	(.0559)	(.0054)	(.9488)	(2.0710)	(1.1776)			
27	9	3	23.1779	.5360	−.2115	−.0041	−.0100	.0293	.0653	−.0129	−1.0316	−1.8461	−.0667	.6562	1.6859	25.82
				(.3289)	(.2224)	(.0539)	(.1796)	(.0252)	(.0614)	(.0087)	(1.3590)	(2.2489)	(1.4962)			
28	10	1	−7.8113	1.1280	.1153	−.0193	.0525	−.0336	.0508	.0006	2.0700	−.9577	−1.0127	.6997	1.3350	25.60
				(.2854)	(.1866)	(.0302)	(.0986)	(.0252)	(.0496)	(.0036)	(1.2893)	(1.5076)	(.9959)			
29	10	2	17.2424	.9220	−.2316	−.0306	−.1274	.0112	.0597	.0043	−.8282	−.0538	1.2322	.6684	1.6187	25.38
				(.3407)	(.1869)	(.0367)	(.1139)	(.0266)	(.0566)	(.0055)	(.9600)	(2.0927)	(1.1894)			
30	10	3	12.7202	.8612	−.2099	−.0594	.0668	.0102	−.0052	.0018	−1.1969	−1.1185	.9638	.6446	1.7140	25.82
				(.3099)	(.2317)	(.0689)	(.1109)	(.0221)	(.0501)	(.0052)	(1.0053)	(1.6412)	(1.4540)			
31	11	2	17.6374	.8937	−.2315	−.0308	−.1186	.0127	.0546	−.0041	−.7956	−.0827	1.2148	.6578	1.6571	25.44
				(.3446)	(.1915)	(.0383)	(.1172)	(.0272)	(.0578)	(.0057)	(.9892)	(2.1424)	(1.2248)			
32	11	3	13.2085	.8658	−.2160	−.0634	.0654	.0103	−.0044	.0020	−1.2342	−1.1866	1.0355	.6412	1.7223	25.82
				(.3184)	(.2328)	(.0691)	(.1114)	(.0223)	(.0503)	(.0052)	(1.0110)	(1.6513)	(1.4612)			
33	12	3	12.3549	.8495	−.2062	−.0572	.0713	.0099	−.0043	.0017	−1.1832	−1.0827	.9667	.6381	1.7607	25.85
				(.3292)	(.2401)	(.0738)	(.1152)	(.0230)	(.0515)	(.0054)	(1.0499)	(1.7367)	(1.5216)			

* For a one-month projection actual lagged price is used.

† The number in parentheses is the standard error of the coefficient.

TABLE 5

EQUATIONS FOR ESTIMATING PRICES OF 700- TO 900- AND 1,100- TO 1,300-POUND STEERS GIVEN AN ESTIMATED PRICE FOR 900- TO 1,100-POUND STEERS

Weight group	Grade	Estimating equation	R_2	$S\hat{y}$
pounds				
700 to 900	Choice	$P^{C}_{7-9} = 1.172 + .960\, P^{C}_{9-11}$	.994	.108
1,100 to 1,300	Choice	$P^{C}_{11-13} = -.742 + 1.019\, P^{C}_{9-11}$	.985	.202
700 to 900	Good	$P^{G}_{7-9} = 1.206 + .959\, P^{G}_{9-11}$	.985	.154
900 to 1,100	Good	$P^{G}_{9-11} = .848 + .922\, P^{C}_{9-11}$	.988	.226
1,100 to 1,300	Good	$P^{G}_{11-13} = -.467 + 1.007\, P^{G}_{9-11}$	.985	.200

Other weights and grades of slaughter cattle

To analyze the decision problems, price estimates are also required for other weights and grades of slaughter cattle as well as for feeder cattle. Analysis of historical data indicates that prices for other weights and grades of slaughter cattle can be derived from the prices for 900- to 1,100-pound Choice grade slaughter steers. The estimated relationships between these various prices are given in table 5. Thus, once the price is predicted for the 900- to 1,100-pound Choice animals, the required prices of other classes can be estimated.

Feeder cattle

Feeder cattle prices were predicted by relating prices of 550 -to 750-pound Good grade feeder steers at Stockton to their lagged values and to predicted slaughter cattle prices. These equations are given in table 6.

The price forecasting model for slaughter steers with the accompanying relations between slaughter cattle prices for other weights and grades provides the basic foundation for marketing the decision problems studied under Models I and II. The feeder cattle price forecasting equations are added in Model III to give the format for purchasing decisions.

PROBABILITY DISTRIBUTIONS

The marketing decision rules developed in this study are based on expected price *changes* rather than expected price *levels,* although the latter also affect the marketing decision. As will be shown later, each price level is considered as a separate decision problem. Therefore, at this point, our concern is with gaining information about price *changes* that may occur.

In the following analysis, two marketing strategies are developed based on probability distributions of historical price data. The first strategy simply uses the marginal probability distribution of historical prices. This distribution is referred to as the "*a priori*" distribution following the terminology of the no data models outlined in the previous section on the decision model.

The second distribution is the conditional probability of a price change given the magnitude of the price change predicted by the price forecasting model. This distribution is hereafter referred to as the "*a posteriori*" distribution which

TABLE 6

PRICE FORECASTING EQUATIONS FOR GOOD GRADE, 550–750 POUND FEEDER STEERS, STOCKTON, CALIFORNIA

Length of price projection	Month of quarter	Constant term a	$\hat{P}P_{(t-1)}$ b_1	$\hat{P}_t$ b_2	Quarterly dummy variables: 1st quarter b_3	2nd quarter b_4	3rd quarter b_5	Shift variable "D" b_6	R^2	$S\hat{y}$
months										*1,000 head*
1	1	1.0770	.7208	.2176	.1501	.1592	−.2946	−1.2268	.9601	.5665
			(.1320)	(.1282)	(.1696)	(.1762)	(.1688)	(.5411)		
1	2	.4930	.5910	.3621	.2033	−.1841	−.4741	−1.5767	.9422	.7094
			(.1507)	(.1172)	(.2040)	(.2172)	(.2024)	(.7014)		
1	3	−1.0475	.7978	.2306	.2733	−.4469	−.1281	−.1661	.9754	.4588
			(.0954)	(.0783)	(.1313)	(.1289)	(.1276)	(.4540)		
2	1	1.4486	.5545	.3595	.1010	.2510	−.3456	−2.0771	.9276	.7692
			(.1629)	(.1639)	(.2321)	(.2460)	(.2284)	(.6960)		
2	2	2.0339	.2671	.6061	.1623	.0074	−.5835	−2.9721	.8982	.9531
			(.1929)	(.1498)	(.2752)	(.3021)	(.2726)	(.9379)		
2	3	−.1371	.4866	.4841	.2553	−.3320	−.1787	−1.6386	.9269	.8033
			(.2175)	(.1726)	(.2310)	(.2356)	(.2245)	(1.0358)		
3	1	1.7004	.3661	.5245	−.0348	.3929	−.4032	−2.8240	.9125	.8593
			(.1768)	(.1840)	(.2532)	(.2696)	(.2681)	(.7620)		
3	2	2.2279	.1181	.7382	.1398	.0924	−.6466	−3.5278	.8751	1.0555
			(.2416)	(.1926)	(.3051)	(.3398)	(.3050)	(1.1267)		
3	3	1.0267	.1923	.7116	.3445	−.3502	−.2181	−3.0114	.8982	.9477
			(.3927)	(.3038)	(.2763)	(.2805)	(.2685)	(1.8341)		
4	1	1.4276	.3690	.5327	−.0624	.4500	−.4383	−2.8438	.9153	.8455
			(.1655)	(.1767)	(.2472)	(.2666)	(.2696)	(.7071)		
4	2	1.9080	.2061	.6684	.1519	.0427	−.6140	−3.2420	.8737	1.0789
			(.2609)	(.2012)	(.3138)	(.3523)	(.3240)	(1.2281)		
4	3	.4558	.1465	.7768	.3408	−.3384	−.2192	−3.3018	.8746	1.0518
			(.4037)	(.3225)	(.3054)	(.3116)	(.2969)	(1.9324)		
5	1	.3739	.3509	.5919	−.0308	.4329	−.4737	−2.9296	.8999	.9188
			(.1702)	(.1859)	(.2725)	(.2895)	(.2880)	(.7244)		
5	2	1.6250	.2732	.6169	.1585	.0606	−.6398	−3.0046	.8710	1.0900
			(.2902)	(.2266)	(.3176)	(.3701)	(.3362)	(1.3445)		
5	3	1.1114	.0648	.8280	.3360	−.3325	−.1894	−3.7607	.8768	1.0585
			(.4545)	(.3593)	(.3109)	(.3168)	(.3159)	(2.1724)		
6	1	.1489	.3751	.5780	−.0105	.4073	−.4729	−2.8134	.9039	.9145
			(.1674)	(.1792)	(.2734)	(.2899)	(.2844)	(.7217)		
6	2	.3891	.3890	.5584	.1547	−.0204	−.5701	−2.7187	.8772	1.0635
			(.4480)	(.3562)	(.3171)	(.4221)	(.3446)	(2.0844)		
6	3	1.8227	−.0621	.9181	.3507	−.2860	−.2386	−4.3962	.8733	1.0731
			(.5117)	(.4012)	(.3162)	(.3251)	(.3257)	(2.4508)		

TABLE 7
POSSIBLE PRICE CHANGES AND VALUES OF Θ USED IN THE STUDY

State	Interval	Value of Θ
	dollars per cwt	
Θ_1	$1.26 \leq \Delta P$	1.50
Θ_2	$.76 \leq \Delta P \geq 1.25$	1.00
Θ_3	$.26 \leq \Delta P \geq .75$	.50
Θ_4	$-.25 \leq \Delta P \geq .25$	.00
Θ_5	$-.75 \leq \Delta P \geq -.26$	−.50
Θ_6	$-1.25 \leq \Delta P \geq -.76$	−1.00
Θ_7	$\Delta P \geq -1.25$	−1.50

provides the basis for the data problem.

Both probability distributions are derived from the same set of price data. To this extent, therefore, one could look upon the *a priori* distribution as a naive price forecasting tool, and the *a posteriori* distribution as being obtained from a more sophisticated predicting mechanism. However, rather than adopting a new terminology at this point, the notation *a priori* and *a posteriori* will be retained in following with the original model formulation.

Average monthly price data for the 1960 to 1969 period (i.e., 120 price observations) are used to calculate the probabilities. Seven ranges of price changes ($\Delta P = P_{(t+1)} - P_t$)[11] are considered in the study shown in table 7. Thus, there are seven points in the sample space over which the probability distributions are to be calculated. Since we are dealing with a discrete set of possible price changes, the probability distributions can be expressed as a table of values. These values (P_i) indicate the probability of a particular value of Θ (price change) occurring and have the following properties (a) $0 \leq P_i \leq 1$ for all i, and (b) $\sum_i P_i = 1$.

The likelihood of a given price change probably is not independent of the current level of prices relative to some "normal" or "average" level of prices. If current prices are high, a large drop in prices is more likely than if current prices were well below average. Therefore, it would be desirable to estimate conditional probability distributions for each price level. However, given data limitations, there are not enough observations in each group to calculate meaningful probabilities for each of the values of Θ. Therefore, the same distribution of price changes is used to represent all price levels.

A priori distribution

The *a priori* probability distribution presents the information about the likelihood of Θ available to the decision maker without any experimentation. It can be derived (either objectively or subjectively) from a historical distribution of Θ.

In this study, the *a priori* distribution is defined as the relative frequency of Θ over the 1960 to 1969 period. Calculation of the *a priori* distribution consists of two steps: (a) calculate $\Theta = \Delta P = P_{(t+1)} - P_t$[12] for all available data and (b) determine the relative frequency with which each value of Θ occurred. For example, table 8 shows that for a one-month price change, $\Theta = 0$ (i.e., $-.25 \leq \Delta P \leq .25$) in 26 out of the 120 months observed and $\Theta = \$1.50$ 13 times. These frequencies are used to construct the probability distributions presented in table 9. On the basis of the distribution of Θ over the 1960 to 1969 period for one-month price changes, we would expect Θ_4 (i.e., $\Theta = 0$) to occur about 22 per cent of the time, Θ_1 (i.e., $\Theta = \$1.50$) to occur about 11 per cent of the time, and so on.

Table 9 summarizes, in objective form, the information about the distri-

[11] t now refers to the current month.

[12] This refers to a one-month price change. For a two-month price change, $\Delta P = (P_{(t+2)} - P_t)$.

TABLE 8

FREQUENCY OF OCCURRENCE OF PRICE CHANGES OVER THE 1960–1969 PERIOD

Period covered	Price change						
	Θ_1 1.50	Θ_2 1.00	Θ_3 0.50	Θ_4 0.00	Θ_5 −0.50	Θ_6 −1.00	Θ_7 −1.50
One-month..........	13	14	21	26	22	12	12
Two-month..........	26	10	21	15	9	11	28

bution of Θ available from the historical record of price changes. With the exception of 1968 and 1969, prices of 900- to 1,100-pound slaughter cattle at El Centro over the period studied fluctuated around an average level of about $25.00. The lack of trend is shown by the expected value of Θ, given the *a priori* probability distribution. The *a priori* expected price change, $E(\Theta) = \sum_j \Theta_j P(\Theta_j)$, is .025 for a one-month price change, and .017 for a two-month price change. Thus, the *a priori* information states that lacking any other information, the cattle feeder reasonably can base his decisions on a zero or very small expected price change.

A posteriori distribution

The *a posteriori* distribution represents the added information from the price forecasting model available to the decision maker at the time the decision is made. Given a predicted price, $\hat{P}_{(t+1)}$, the experiment consists of calculating $Z = \hat{P}_{(t+1)} - P_t$. Z functions as an estimator of Θ for the coming month(s).[13] The accuracy of Z as a predictor of Θ depends on the accuracy of the price forecasting model.

The *a posteriori* probability distribution $P(\Theta|Z)$ yields the probability that a particular value of Θ (price change) will occur, given Z as a prediction of Θ. As explained earlier, the conditional probability of $P(\Theta|Z)$ can be calculated by Bayes' formula when the *a priori* distribution is developed separately from the *a posteriori* distribution. If the data set for both distributions is identical, as is the case here, then the conditional distribution of $\Theta|Z$ can be calculated directly without the use of Bayes' formula. As long as the observations are the same, then the direct approach and the Bayesian formulation will yield identical answers.

Bayes' formula, however, permits the use of additional information which may be available for the estimation of one probability distribution (say $P(\Theta)$) but not for the direct estimation of the conditional distribution. Thus, a feeder could substitute his own *a priori* distribution into the above analysis through use of Bayes' formula should he so desire.

TABLE 9

A PRIORI PROBABILITY OF PRICE CHANGES

Period covered	Price change						
	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5	Θ_6	Θ_7
One-month..........	.108	.117	.175	.217	.183	.100	.100
Two-month..........	.217	.083	.175	.125	.075	.092	.233

[13] For two-month estimates of Θ, Z is calculated as $(\hat{P}_{(t+2)} - P_t)$.

TABLE 10

FREQUENCY WITH WHICH Z_k WAS PREDICTED GIVEN THAT Θ_j WAS THE TRUE STATE OF NATURE

Actual price change	Predicted price change							
	Z_1 1.50	Z_2 1.00	Z_3 0.50	Z_4 0.00	Z_5 −0.50	Z_6 −1.50	Z_7 −1.50	Total
$\Theta_1 =$ 1.50	4	6	2	1				13
$\Theta_2 =$ 1.00	1	4	6	3				14
$\Theta_3 =$ 0.50	1	6	5	5	3			20
$\Theta_4 =$ 0.00			7	8	8	3		26
$\Theta_5 =$ −0.50			2	9	9	2		22
$\Theta_6 =$ −1.00				3	4	4	1	12
$\Theta_7 =$ −1.50				2	3	4	3	12
Total	6	16	22	31	27	13	4	119

$P(\Theta|Z)$ is obtained by evaluating the performance of Z as a predictor of Θ over the historical period. That is, using the predicted values generated by the forecasting model and actual historical data, we can construct a table showing the frequency with which actual price changes, Θ, took on certain values, given alternative predicted values, Z. This information is presented in table 10 for one-month price changes over the 1960 through 1969 period. For example, during this period $Z_4(-25 \leq \Delta P \leq .25)$ occurred 31 times; the predicted intervals were correct eight times, too high 14 times and too low nine times. These frequencies are used to determine the conditional probabilities shown in table 11.

The *a posteriori* distribution is also needed for two-month price changes. The procedure used to calculate this distribution is the same as that for the one-month price change with one modification; the price intervals used to estimate the probabilities are increased to one-dollar magnitudes rather than 50 cents. This change results from the increased frequency of larger price changes as the time period increases.

Therefore, the number of possible price changes (defined as ϕ for Model II) are as follows:

TABLE 11

A POSTERIORI DISTRIBUTION $P(\Theta|Z) = \dfrac{P(Z|\Theta)\ (P\Theta)}{PZ}$ OF ONE MONTH PRICE CHANGES

Actual price change	Projected price change						
	Z_1 1.50	Z_2 1.00	Z_3 0.50	Z_4 0.00	Z_5 −0.50	Z_6 −1.00	Z_7 −1.50
$\Theta_1 =$ 1.50	.666	.375	.091	.032			
$\Theta_2 =$ 1.00	.167	.250	.273	.097			
$\Theta_3 =$ 0.50	.167	.375	.227	.161	.111		
$\Theta_4 =$ 0.00			.318	.258	.296	.230	
$\Theta_5 =$ −0.50			.091	.290	.334	.154	
$\Theta_6 =$ −1.00				.097	.148	.308	.250
$\Theta_7 =$ −1.50				.065	.111	.308	.750
$P(Z)$	.050	.136	.185	.261	.226	.108	.034

	Interval	*Value of ϕ_i*
ϕ_1	$\$ .51 \leq \Delta P \leq \$ 1.50$	$\$ 1.00
ϕ_2	$\$- .50 \leq \Delta P \leq \$ .50$	0
ϕ_3	$\$-1.50 \leq \Delta P \leq \$- .51$	-1.00

The two extreme intervals (ϕ_1 and ϕ_3) are actually open-ended in the sense that they include price changes of magnitudes greater in absolute terms than \$1.50. The use of a representative value for ϕ_i generally does not affect the analysis; however, some degree of caution is needed in making decisions incorporating the price forecasting model when the price forecast is for a price decrease of more than \$1.50. This factor will be discussed later. The *a priori* and *a posteriori* distributions for Model II are presented in table 12.

TABLE 12
PROBABILITY DISTRIBUTIONS FOR TWO-MONTH PRICE CHANGES

Actual price change	*A posteriori* distribution $P(\phi \mid Z)$ — Predicted price change: Z_1 1.00	Z_2 0.00	Z_3 −1.00	*A priori* distribution $P(\phi)$
ϕ_1 = 1.00....	.814	.296	.021	.383
ϕ_2 = 0.00....	.186	.333	.250	.242
ϕ_3 = −1.00....	.000	.371	.729	.375

FEEDLOT PERFORMANCE OF CATTLE

Two additional sets of information are required for the decision models: (a) the cost per pound of gain and (b) the expected carcass grade at alternative slaughter weights. In the general formulation of the decision problems, these two factors could be incorporated as additional states of nature, W and U. However, most experienced cattle feeders can estimate accurately the values of these variables. In this analysis, we assume that the errors associated with the feeder's estimates are distributed about the true value with a very small variance and that the errors, therefore, are negligible. These relationships have been incorporated into the decision models at their average (expected) value. To the degree that these assumptions are not met, a greater needed expected price increase than suggested by the following model would be required. The situation will be discussed later in the section on sensitivity of the model.

Estimation of cost per pound of gain

Two sets of cost estimates are needed: (a) an estimate of the average cost per pound of gain over the normal feeding period (defined here as feeding 600-pound feeder steers to 1,000-pound slaughter weight) in order to calculate break-even feeding margins for the purchasing decisions and (b) break-down of cost per pound of gain as the weight of the animal progresses from 600 to 1,200 pounds—for example, an estimate of the cost per pound of gain for feeding 950-pound steers another 30 days vs the cost per pound of gain for feeding 1,100-pound steers another 30 days. Since the cost per pound of gain increases with the weight of the animal, specific information about this relationship is needed to make marketing decisions for alternative weights of cattle. The information is needed for weights up to 1,200 pounds, since marketing decisions generally are made for slaughter cattle ranging up to this weight.

The derivation of these estimates consists of (a) specifying a typical finishing ration and determining its net energy content, (b) estimating daily feed consumption for various weight groups, (c)c salculating the resulting gain per day for each weight group using the tables in Lofgreen and Garrett (1968), and (d) calculating the feed cost per pound of gain, using average feed prices. Nonfeed costs are then added to feed costs to arrive at total cost per pound of gain.

Ration. The Hopkin and Kramer (1965) survey indicates the average finishing ration for cattle feedlots in California has the following composition: roughages, 15.3 per cent; feed grains, 57.6 per cent; other high-energy feeds, 14.5 per cent; protein supplement, 3.7 per cent; molasses, 5.7 per cent; fat, 2.1 per cent; and minerals, 1.1 per cent. Alfalfa, barley and milo, beet pulp, and cottonseed meal were the most widely used feeds in their respective categories. Using this information, the following ration was formulated as the "typical" finishing ration.[14]

Feed	*Percentage of total ration*
Alfalfa	15.3
Barley	42.3
Milo	15.3
Beet pulp	14.5
Cottonseed meal	3.7
Molasses	5.7
Fat	2.1
Minerals	1.1
TOTAL	100.0

This ration provides 79.95 megcal. of net energy for maintenance (NE_m) and 50.69 megcal. of net energy for production (NE_p) per 100 pounds of feed.[15] It also provides all of the nutrient requirements, except vitamin A for fattening yearling cattle, as specified by Morrison (1952). The latter requirement can easily be met by adding a vitamin supplement to the ration. The Morrison standards and the nutrient levels supplied by this ration are shown in tables 13 and 14.

Feed consumption. The National Academy of Sciences, NRC (1963) report on nutrient requirements for beef cattle indicates that yearling cattle on a finishing ration consume from 17.5 pounds of feed per day at 600 pounds body weight to 25.8 pounds at 1,100 pounds. These intake levels range from 2.9 percent to 2.3 percent of body weight. The report further states, "Finishing cattle consume feeds in amounts equal to 2.2 to 3.0 per cent of their live weight, dependent upon the concentrate-roughage ratio and age and condition of animals. Older cattle and more fleshy individuals consume less feed per unit of body weight than do younger animals carrying less condition . . . As concentrate content increases, feed consumption is usually reduced."

The following "rule of thumb" for determining feed consumption appears to be consistent with the NRC statement.

Percentage of concentrate in ration	*Daily feed consumption as percentage of live weight*
90	2.4 to 2.6
80 to 90	2.6 to 2.7
70 to 80	2.7 to 2.8
< 70	2.8
All roughage	2.9

[14] The ration used here can be varied to achieve different rates of gain or to take advantage of changes in price relationships among the various feed components. Since many feedlots carry sizable storage of grain and hay (King, 1962), however, the specification of a particular ration for a period of one month does not appreciably affect the nature of the problem solution.

[15] The net energy characteristics of a ration are determined by the amount of each feed used in the ration. Each feed has a given net energy content for maintenance and for gain. The net energy figures used in these calculations were obtained from Lofgreen and Garrett (1968, pp. 24–25).

TABLE 13

MORRISON FEEDING STANDARDS: PER HEAD DAILY NUTRIENT REQUIREMENT FOR FATTENING YEARLING CATTLE

Weight of animal	Dry matter	Digestible protein	Total digestible nutrients	Calcium	Phosphorus	Carotene
	pounds					*mg*
600	15.0–17.6	1.18–1.32	10.7–12.3	.044	.037	35
700	16.5–19.1	1.36–1.52	12.7–14.3	.044	.040	40
800	17.8–20.4	1.52–1.68	14.1–15.9	.044	.042	45
900	18.9–21.7	1.64–1.82	15.4–17.2	.044	.044	50
1,000	20.0–23.0	1.71–1.91	16.0–18.0	.044	.044	55
1,100	21.0–24.0	1.76–1.96	16.5–18.5	.044	.044	60

TABLE 14

NUTRIENTS SUPPLIED BY "TYPICAL" RATION: BASED ON DAILY FEED INTAKE SHOWN IN TABLE 15

Weight of animal	Dry matter	Digestible protein	Total digestible nutrients	Calcium	Phosphorus	Carotene
	pounds					*mg.*
600	14.53	1.34	11.67	.064	.046	23.06
700	16.35	1.51	13.13	.072	.052	25.94
800	17.99	1.66	14.45	.080	.057	28.55
900	19.46	1.80	15.63	.086	.062	30.89
1,000	20.76	1.92	16.67	.092	.066	32.94
1,100	21.88	2.02	17.57	.097	.070	34.73

These two sources of information are combined into three constraints for estimating daily feed consumption: (a) daily consumption rates vary from 2.2 to 3.0 per cent of body weight; (b) daily feed consumption as percentage of body weight declines as liveweight increases; and (c) average consumption rate over the normal feeding period (600 to 1,000 pounds) should average about 2.6 per cent of body weight, given the above 85 per cent concentrate ration.

In this study, a consumption rate of 2.6 per cent was assigned to 800-pound animals (the midpoint for the 600- to 1,000-pound normal feeding interval); then linear interpolation consistent with constraints 1 and 2 was used to estimate consumption rates for the remaining weight groups. The consumption rates are then applied to the liveweight to arrive at estimates of daily feed consumption shown in column 3 of table 15.

These consumption estimates are slightly below those presented in the NRC report. However, since the ration used here is a higher energy ration than the NRC ration, the results appear to be consistent.

Daily gain. The daily intake of net energy and the resulting gain per day were calculated using the tables in Lofgreen and Garrett (1968). The average daily gain from 600 pounds to 1,000 pounds is 2.77 pounds. This level again is slightly higher than those reported in the NRC report; however, it seems reasonable, given the higher energy content of the ration being considered.

The average daily gains shown in table 15 compare favorably with the results of a survey reported by Logan and King (1966) in which animals placed on feed weighing 600 to 799 pounds had an

Table 15

RESULTS OF FEEDING "TYPICAL" RATION, ASSUMING A NORMAL FEEDING PERIOD OF 600 TO 1,000 POUNDS

Weight of feeder steer	Daily feed consumption as percentage of body weight	Feed consumption per day	Daily maintenance requirements NE_m	Feed per day required for maintenance*	Feed available per day for gain	NE_p† available per day	Daily gain	Feed per pound of gain	Feed cost per pound of gain
pounds	*per cent*	*pounds*	*megcal*	*pounds*	*pounds*	*megcal*	*pounds*	*pounds*	*cents*
600	2.80	16.80	5.21	6.52	10.28	5.21	2.80	6.00	15.53
650	2.75	17.88	5.53	6.92	10.96	5.56	2.81	6.36	16.46
700	2.70	18.90	5.85	7.32	11.58	5.87	2.80	6.75	17.47
750	2.65	19.88	6.16	7.71	12.17	6.17	2.80	7.10	18.37
800	2.60	20.80	6.47	8.10	12.70	6.44	2.79	7.46	19.31
850	2.55	21.68	6.77	8.47	13.21	6.70	2.77	7.83	20.26
900	2.50	22.50	7.06	8.83	13.67	6.93	2.75	8.18	21.17
950	2.45	23.28	7.36	9.21	14.07	7.13	2.72	8.56	22.15
1,000	2.40	24.00	7.65	9.57	14.43	7.31	2.69	8.92	23.08
1,050	2.35	24.68	7.93	9.92	14.76	7.48	2.65	9.31	24.09
1,100	2.30	25.30	8.21	10.27	15.03	7.62	2.61	9.69	25.08
1,150	2.25	25.88	8.49‡	10.62	15.26	7.74	2.57	10.07	26.06
1,200	2.20	26.40	8.77‡	10.97	15.43	7.82	2.52	10.48	27.12

* $NE_m \div .7994$.
† (.5069) (feed available for gain).
‡ The Net Energy requirement for steers over 1,100 pounds is not presented in Lofgreen and Garrett (1968). The values shown here were supplied by G. P. Lofgreen.

average daily gain of 2.68 pounds per day over the feeding period. If the average daily gain from table 15 is extended to cover weights above 1,000 pounds, the estimated gains correspond even closer with the survey results.

Cost per pound of gain. The typical ration cost $2.588 per 100 pounds at average feed prices for 1967, yielding a feed cost per pound of gain from 15.33 cents per pound for 600-pound steers to 27.12 cents per pound for 1,200-pound steers. Feed cost per pound of gain averages 19.31 cents per pound over the 600- to 1,000-pound feeding range.

King (1962, p. 30) has estimated nonfeed costs of 6.46 cents per head per day for feedlots with capacity of 11,280 head operating at 80 per cent capacity.[16] If cattle gain 2.77 pounds per day, the normal feeding period (600 to 1,000 pounds) would be 144 days, giving a nonfeed cost per pound of gain over the normal feeding period of 2.31 cents. Thus, the estimated average total cost of gain over the 600- to 1,000-pound feeding interval is 21.62 cents per pound. This estimate of total cost per pound of gain is used to calculate break-even margins for the purchasing decisions, because we assume that these decisions are based on expected returns from a normal feeding period. To the extent that cattle are fed to weights above 1,000 pounds, this estimate will be biased downward.

Carcass grade—Slaughter weight relationship

Prices of Choice grade slaughter steers are generally above Good grade prices. Therefore, the carcass grade that feeder steers can be expected to attain at the end of the feeding period is an important component of purchasing and marketing decisions. The expected slaughter grade clearly affects the expected feeding margin and, therefore, the price the cattle feeder can afford to pay for feeder cattle. Similarly, the extent of upgrading achieved by feeding another 30 days directly affects expected returns from extending the feeding period.

Carcass grade is related to the conformation of the animal and the degree of finish. Some animals with good conformation may grade Choice at 700 pounds or perhaps less; other animals may not attain Choice grade even if fed to 1,200 pounds. Conversely, conformation alone is not sufficient for grading Choice.

Given steers of a specified age and conformation, the expected carcass grade is a function of the weight at which the animals are slaughtered. Slaughter weight serves as an indicator of finish for steers of similar conformation. Finish refers to a combination of fat covering and marbling (i.e., presence of fat in the muscling) and is not a linear function of weight. The proportion of additional weight added as fat covering (relative to additional marbling) increases as the weight of the animal increases. Beyond a minimal standard added fat covering does little to improve the grade of the carcass. Therefore, we would expect the probability of attaining Choice grade to increase at a decreasing rate as the weight of the animal increases.

At any given weight of an animal, the probability of its attaining a certain grade can be considered as discrete, that is, at each possible weight the steer will either grade less than Choice or will grade Choice or better. On the other hand, the relationship between grade

[16] A capacity of 11,280 head is assumed to be a representative size feedlot. Cost figures for 80 per cent capacity are used because published data indicate only about 75 per cent of the total feedlot capacity in California is utilized. The nonfeed costs include management and office, taxes, insurance, interest, depreciation, death loss, labor, and utilities.

and weight can be expressed as a continuous function. By expressing the percentage grading Choice or better as a function of average weight of the lot, a continuous relationship between carcass grade and slaughter weight can be derived. Because it is a continuous function, we have a measure of the degree of upgrading that can be expected from feeding cattle from one weight to another as well as the percentage of animals that can be expected to grade Choice or better at alternative weights.

The weight-grade relationship is based on 2,080 observations of slaughter weight and carcass grades of cattle. The data consist of 487 observations from cattle feeding experiments performed by the University of California and 1,593 observations from two northern California slaughter plants.[17] The data obtained from the slaughter plants were in terms of *carcass* weight and grade. Carcass weights were converted to live weights by using the following dressing percentages:

Carcass weight	*Dressing percentage*
< 550 pounds	59.5
551 – 600 pounds	60.0
> 600 pounds	60.5

For example, a 580-pound carcass weight is converted to live weight as follows: $(580 \div .60) = 967$ pounds. The slaughter weights were grouped into 50-pound intervals with the midpoint for the interval ranging from 725 pounds to 1,275 pounds. The percentage of cattle grading Choice or better was then calculated for each weight interval and plotted against the average weight of the interval (figure 1). If the resulting live weight calculated from carcass data was with ± 2 pounds of the interval boundaries, the observation was discarded to reduce errors of placing a carcass in the wrong interval.

The sample data represented by dots in figure 1 show that the percentage grading Choice increases rather rapidly from 725 pounds to about 1,000 pounds and levels off beyond 1,000 pounds. The data also indicate that the function is asymptotic at some value less than 100. That is, some of the cattle fed will never attain Choice grade regardless of weight.

Two simple equational forms were fitted to the data: (a) $Y = a + b\sqrt{X}$, and (b) $Y = a + \log_e X$; where Y is the percentage grading Choice or better and X is the *average weight* of cattle in each weight interval. In both cases the resulting equations are essentially linear over the range of data because of the rather large magnitude and relatively narrow range of the values of X (weights). However, this nonlinear relationship in the square root and semilog functions is most prevalent when the values of X are small and there is a relatively wide range of X values considered. For large numbers, $\sqrt{X}$ and $\log_e X$ approach linearity, particularly over short ranges.

To avoid this linearity, the scale of the X axis was changed to reduce the absolute magnitude of the X values and thus increase the *relative* range of the dependent variable. This was accomplished by forming a new variable $Z = -13.5 + .02X$. For example, if $X = 725$, $Z = -13.5 + .02(725) = 1$. If $X = 775$, $Z = -13.5 + .02(775) = 2$, and so

[17] The use of these data to estimate the relationship between carcass grade and slaughter weight requires two major assumptions regarding the homogeneity of the data both within and among the two sets: (1) that the animals observed were of similar quality when placed on feed and are representative of the quality of cattle fed in California, and (2) that the feedlots in which these cattle were fed are typical of California feedlots in feeding programs followed and rations fed. We have no information to substantiate or repudiate either of these assumptions.

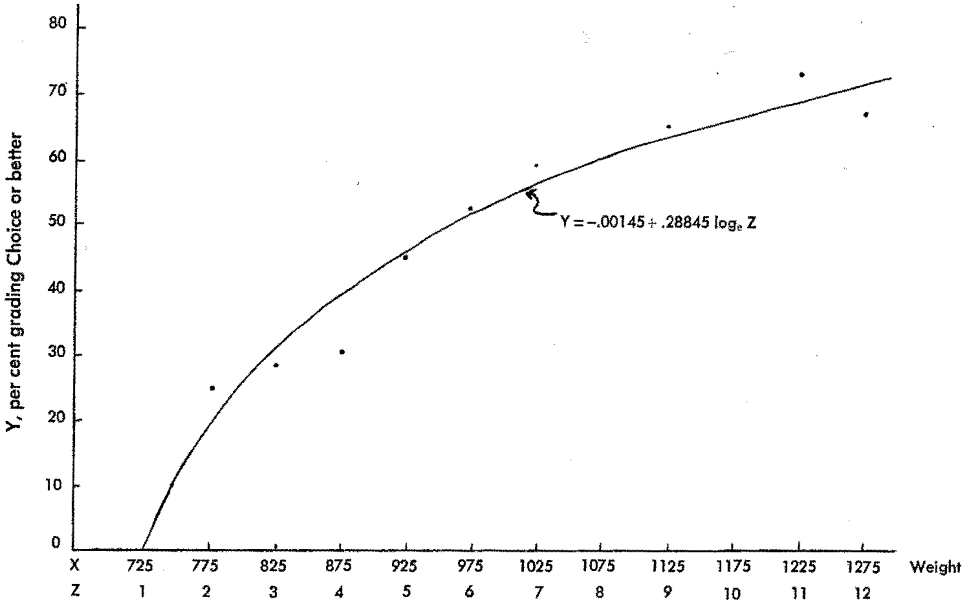

Fig. 1. Relationship between slaughter weight and carcass grade.

forth. Y can now be expressed as a function of Z instead of the original variable X.

Both the square root and semi-log functions were fitted to the adjusted data. The semi-log function has the highest degree of explanation and lowest standard error. The resulting equation is as follows:[18]

$$Y = -.00145 + \underset{(.21635)^*}{.28845} \log_e Z$$

$$R^2 = .969 \tag{11}$$

$$S_{\hat{y}} = .041$$

where: Y is the percentage grading Choice or better.
$Z = -13.5 + .02X$; X is the average slaughter weight of cattle in the lot.
$*$ = standard error of coefficient.

This equation is plotted against actual observations in figure 1.

Using equation 11, we can obtain an estimate of the percentage of animals in a particular lot that can be expected to grade Choice or better, given the average weight of the cattle in the lot. For example, if the average weight of a lot of cattle is 1,050 pounds, the expected percentage grading Choice is estimated as follows:

$$Z = -13.5 + .02(1{,}050) = 7.50$$

$$\log_e Z = 2.0149$$

$$Y = -.00145 + .28845(2.01490) = 58.0 \text{ per cent.}$$

The relationship estimated here is consistent with information provided by the Hopkin-Kramer survey (1965) in which 58 per cent of the fed cattle marketed in California in 1963 were Choice grade.

[18] The square root function is: $Y = -.19312 + \underset{(.02101)^*}{.27575} \sqrt{Z}$ $R^2 = .945$ $S_y = .054$; where Y and Z are defined above. *Indicates standard error of coefficient.

DECISION RULES

Four models are developed in this section. As stated before, model I uses Bayesian decision theory to arrive at a set of short-run marketing decision rules based on a one-month prediction of slaughter cattle prices. Model II extends the marketing decision rules to cover two-month price predictions. Model III incorporates the information provided by the price forecasting model to develop criteria for purchasing feeder cattle. Model IV combines the marketing and purchasing decision rules into a six-month planning model.

Feeding decisions for one month—Model I

The decision to feed a particular lot of cattle another 30 days or sell them at current weight is based on *expected* profit from extending the feeding period.[19] A particular lot of cattle will be continued on feed only if expected marginal profit from additional feeding is positive.

The outcome, λ (marginal profit per head) of extending the feeding period another 30 days is defined by equation (12).

$$\lambda = GV - [CV + C]$$

$$\lambda = P_{(t+1)}(W + G) - (P_t W + C) \quad (12)$$

$$\lambda = P_{(t+1)}G + \Delta PW - C$$

$$\lambda = (\Delta P + P_t)G + \Delta PW - C$$

where:

GV = gross value of steer 30 days hence (dollars per head)
CV = current value of steer (dollars per head)
C = cost of feeding the steer another 30 days (dollars per head)
P_t = current price (dollars per pound)
$P_{(t+1)}$ = price 30 days hence (dollars per pound)
ΔP = $(P_{(t+1)} - P_t)$ = price change during next 30 days (dollars per pound)
W = current weight of steer (pounds)
G = gain from feeding another 30 days (pounds)

Thus, the marginal profit from extending the feeding period another 30 days is the value of the weight gained $(\Delta P + P_t)G$, plus the change in value of the current weight (ΔPW), minus feeding cost.

As shown previously, daily gain and feeding cost both are a function of the weight of the steer. Consequently, the outcome of extending the feeding period can be expressed as a function of (a) current weight of the steer, (b) current slaughter cattle price, and (c) the price change that occurs. Current weight and current slaughter cattle prices are known at the time the decision is made to sell or continue feeding. Thus, the uncertainty about the outcome of a decision to extend the feeding period arises from the price change that may occur.

The price change, ΔP, has two components: the change in the level of slaughter cattle prices and an increase in price received resulting from upgrading. The upgrading has the effect of increasing ΔP since prices of Choice grade slaughter cattle average about one dollar per hundredweight above Good grade prices. Thus, in evaluating equation (12), explicit attention must be given to the degree of upgrading that

[19] The decision rules are based on expected returns to the "average" steer in a particular lot of cattle. The rules are applicable to marketing and purchasing decisions for the entire lot of cattle of specified average weight rather than to individual animals.

can be expected by incorporating the weight-slaughter grade relationship expressed by equation (11).

The Bayesian statement of the decision problem is as follows: Given the average weight of a particular lot of cattle and the current price of slaughter cattle, the cattle feeder is faced with two possible actions—A_1 = feed the cattle another 30 days or A_2 = sell the cattle at current weight. The outcome (λ_{ij}) of these decisions depends on the change in the price of slaughter cattle (state of nature Θ_j) that occurs in the next 30 days. Using the range of price changes (states of nature) defined in table 14, the decision problem can be formulated as follows:

Actions	States of nature $\Theta = [P_{(t+1)} - P_t]$ (dollars)						
	Θ_1 1.50	Θ_2 1.00	Θ_3 0.50	Θ_4 0.00	Θ_5 −0.50	Θ_6 −1.00	Θ_7 −1.50
A_1 (feed)	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}
A_2 (sell)	λ_{21}	λ_{22}	λ_{23}	λ_{24}	λ_{25}	λ_{26}	λ_{27}

λ_{ij} is the outcome (marginal profit) of following action A_i, given the occurrence of Θ_j. $\lambda_{2j} = 0$ for all values of j, because marginal profits from selling at current weight are zero. $\lambda_{1j} \lesseqgtr 0$, depending on the price change that occurs. The cost of feeding, C, is the same regardless of which value of Θ that occurs.

The "no data" strategy is derived using the *a priori* distribution, $P(\Theta)$, and is defined as selecting the action for which $\hat{\pi}_i = \sum_j \lambda_{ij}P(\Theta_j)$ is the greatest, where $\hat{\pi}_i$ is the expected profit of the i^{th} action.[20] The "data" strategy utilizes the *a posteriori* distribution, and the optimal action is the one for which $\hat{\pi}_i = \sum_j \lambda_{ij}P(\Theta_j|Z_k)$ is the maximum, where Z_k is the predicted price change.

Derivation of decision rules. Current price and current weight are held constant in the above formulation of the problem; however, these factors directly affect the outcome of the decision (equation 12). Therefore, decision rules must be derived for alternative combinations of prices and weights. Nine weight categories and eight price levels are considered, as shown in table 16. Thus, 72 separate decision problems are considered, one for each combination of weight and current prices.

The price levels in table 16 are for 900- to 1,100-pound Choice grade slaughter steers. *All statements regarding price levels and price changes refer to prices of 900- to 1,100-pound Choice grade steers.* By using the price relationships presented in table 5 to evaluate the outcome of each action, we can state the resulting decision rules in terms of current levels and price changes for 900- to 1,100-pound Choice grade slaughter steers.

The first step in developing the decision rules is to evaluate the outcome, λ_{ij}, of each action under alternative states of nature (price changes). The second step is to calculate the *expected* profit from each action utilizing the *a priori* and *a posteriori* probability distributions. Since $\lambda_{2j} = 0$ (i.e., the outcome

[20] Throughout this study we are assuming that the cattle feeder's utility function is linear with respect to money over the relevant range. Thus decision rules are developed by maximizing expected profits which is equivalent to maximizing expected utility.

TABLE 16

WEIGHT CATEGORIES AND PRICE LEVELS FOR WHICH DECISION RULES ARE CALCULATED

Current weight*		Current price, 900- to 1100-pound choice grade slaughter steers	
Weight range	Representative weight	Price range	Representative price
pound		*dollars per cwt*	
775– 825	800	20.50–21.50	21.00
826– 875	850	21.51–22.50	22.00
876– 925	900	22.51–23.50	23.00
926– 975	950	23.51–24.50	24.00
976–1,025	1,000	24.51–25.50	25.00
1,026–1,075	1,050	25.51–26.50	26.00
1,076–1,125	1,100	26.51–27.50	27.00
1,126–1,175	1,150	27.51–28.50	28.00
1,176–1,225	1,200		

* Average weight of cattle in a particular lot.

of the sell action) for each state of nature, we only have to evaluate the possible outcomes of extending the feeding period another 30 days for alternative weight groups and price levels.

The data needed to evaluate the outcomes (equation 12) are presented in table 17. Evaluation of equation (12) under alternative price changes consists of three steps: (1) calculate the gross value at the end of the extended feeding period, (2) calculate current value plus cost of feeding another 30 days, and (3) calculate $\lambda = GV - [CV + C]$.

The average gross value per head at the end of the extended feeding period is defined as

$$GV = W_2[rP^C + (1 - r)P^G]\ ^{21}$$

TABLE 17

DATA NEEDED TO ESTIMATE GROSS VALUE OF STEERS AT END OF EXTENDED FEEDING PERIOD AT ALTERNATIVE PRICES

Weight of cattle		Cost of feeding another 30 days†	Proportion of cattle grading Choice at ‡	
Current	30 days hence*		Current weight	End of extended feeding period
pounds		*dollars/head*	*per cent*	
800	884	18.16	.263	.411
850	933	18.76	.360	.472
900	983	19.51	.432	.523
950	1,032	20.10	.490	.566
1,000	1,081	20.63	.538	.602
1,050	1,130	21.20	.580	.636
1,100	1,178	21.50	.616	.666
1,150	1,227	22.01	.648	.690
1,200	1,276	22.55	.677	.715

* Computed as current weight plus daily gain shown in table 15 times 30.
† Cost per pound of gain shown in table 15 times weight gained plus $1.94 = (30)(.0646) nonfeed cost.
‡ Calculated using equation (11).

[21] The decision rules developed here are based on the expected returns per head for a lot of cattle with specified average weight. Therefore, the prices used to calculate expected returns are a weighted average of Choice and Good grade prices. The proportion of the cattle grading Choice at alternative weights is defined by equation (11).

where

W_2 = weight at end of extended feeding period (100 pounds),

r = proportion of cattle grading Choice, and

P^C and P^G = price of Choice grade and Good grade slaughter steers, respectively, at the end of the extended feeding period (dollars/cwt).

The appropriate values of W_2 and r for each weight group are shown in table 17. Thus, given that the current price is \$23.00 and increases \$0.50 over the feeding period, the gross value *at the end of the extended feeding period* for a steer currently weighing 850 pounds is

$$GV = 9.33[.472(23.50) + .528(22.52)] = \$214.42.$$

Similarly, given the same price conditions, the gross value at the end of the extended feeding period for a steer currently weighing 1,150 pounds is

$$GV = 12.27[.690(23.20) + .310(22.20)] = \$280.86.$$[22]

The current value of a steer is the gross value of the steer at current weight and prices. For example, the average current value per head of 850-pound steers when the base price is \$23.00 is

$$CV = 8.50[.360(23.25) + .640(22.36)] = \$192.78.$$

The current value of 1,150-pound steers is

$$CV = 11.50[.648(22.70) + .352(21.74)] = \$257.16.$$

The cost of feeding steers of alternative weights is shown also in table 17. Thus the outcomes (marginal profit per head) of feeding 850-pound and 1,150-pound steers another 30 days when current prices are \$23.00 and prices increase \$0.50 over the period are calculated as follows:

850-pound steers:

$$\lambda = 214.42 - [192.78 + 18.76] = \$2.88$$

and

1,150-pound steers:

$$\lambda = 280.86 - [257.16 + 22.01] = \$1.69.$$

The outcomes of feeding other weights under alternative price conditions are calculated by the same procedure. For example, the outcomes of extending the feeding period another 30 days for 900-pound steers under alternative price conditions are presented in table 18.

The next step is to calculate the expected profit of each action under alternative conditions and select the optimal action for each situation.

No data strategies. The expected profit from action A_i is defined as $\pi_i = \sum_j \lambda_{ij} P(\Theta_j)$. For example, the expected profit from feeding 900-pound steers another 30 days when the current price is \$23.00 is, $\pi = .108(14.21) + .117(9.48) + .175(4.76) + .217(-.02) + .183(-4.74) + .100(-9.47) + .100(-14.20) = \$.24$. The expected profits, based on *a priori* information, for actions A_1 and A_2 relating to 900-pound steers and alternative price levels are shown in table 19. The optimal action (greatest expected profit) for each price level is shown in the last column of table 19. The "no data" strategy for ex-

[22] The prices of 1,227-pound steers are keyed to prices of 900- to 1,100-pound Choice grade steers as indicated above. Therefore, the value of the steer at the end of the feeding period is based on a price of \$23.50 for Choice grade 900- to 1,100-pound steers.

TABLE 18

OUTCOMES, λ_{1j}, OF EXTENDING THE FEEDING PERIOD 30 DAYS FOR 900-POUND STEERS UNDER ALTERNATIVE PRICE SITUATIONS

Current price	Price change during extension of feeding period						
	Θ_1 1.50	Θ_2 1.00	Θ_3 0.50	Θ_4 0.00	Θ_5 −0.50	Θ_6 −1.00	Θ_7 −1.50
dollars	*dollars/cwt*						
21.00	12.44	7.71	2.89	−1.74	−6.47	−11.20	−15.93
22.00	13.35	8.57	3.84	− .88	−5.61	−10.34	−15.06
23.00	14.21	9.48	4.76	− .02	−4.74	− 9.47	−14.20
24.00	15.02	10.30	5.57	.84	−3.88	− 8.66	−13.39
25.00	15.89	11.16	6.43	1.71	−3.02	− 7.75	−12.48
26.00	16.75	12.02	7.30	2.57	−2.16	− 6.88	−11.61
27.00	17.62	12.89	8.16	3.43	−1.29	− 6.02	−10.75
28.00	18.53	13.80	9.02	4.30	− .43	− 5.16	− 9.88

tending the feeding period of 900-pound steers can be summarized as follows: Feed if $P \geq \$23.00$; Sell if $P < \$23.00$, where P refers to the current price of 900- to 1,100-pound Choice grade steers.

The "no data" strategies for other weights of cattle are calculated by the same procedure and are summarized in table 20. For example, the no data strategy is to extend the feeding period of 1,000-pound steers only if current prices exceed $25.00. Generally as the animal's weight increases, higher current prices are required to continue feed, a factor reflecting increasing costs of gain. Based on the historical distribution of price changes over the 1960–1969 period, the expected margin of profit from extending the feeding period of cattle weighing 1,050 pounds is negative regardless of current prices. This result occurs because the weight of the animals 30 days hence will put them in the 1,100- to 1,300-pound price class which typically is lower than the 900- to 1,100-pound category.

The "no data" decision rules summarized in table 20 are appropriate if the cattle feeder is willing to base his expectations about future price changes solely on the frequency distribution of these changes in the past. This would be reasonable if (a) no other information about possible price changes were avail-

TABLE 19

A PRIORI EXPECTED PROFITS FOR ACTIONS A_1 AND A_2 AT ALTERNATIVE PRICE LEVELS: 900-POUND STEERS

Current price	Expected profit		Optimal action
	A_1 = feed	A_2 = sell	
dollars/cwt	*dollars/head*		
21.00	−1.51	0	Sell
22.00	− .64	0	Sell
23.00	.24	0	Feed
24.00	1.07	0	Feed
25.00	1.94	0	Feed
26.00	2.81	0	Feed
27.00	3.67	0	Feed
28.00	4.54	0	Feed

TABLE 20

BAYESIAN "NO DATA" STRATEGIES, BASED ON *A PRIORI* PROBABILITY DISTRIBUTION

Current weight	"No data" strategy
pounds	
800	Feed regardless of current price
850	Feed if $P \geq 25.00$; Sell if $P < 25.00$*
900	Feed if $P \geq 23.00$; Sell if $P < 23.00$
950	Feed if $P \geq 24.00$; Sell if $P < 24.00$
1,000	Feed if $P \geq 25.00$; Sell if $P < 25.00$
1,050	Sell regardless of current price
1,100	Feed if $P \geq 27.00$; Sell if $P < 27.00$
1,150	Feed if $P \geq 28.00$; Sell if $P < 28.00$
1,200	Sell regardless of current price

* P refers to the current price of 900- to 1,100-pound Choice grade slaughter steers (dollars/cwt).

TABLE 21

EXPECTED PROFITS PER HEAD FROM EXTENDING THE FEEDING PERIOD 30 DAYS FOR 900-POUND STEERS AT ALTERNATIVE LEVELS OF PRICES AND PREDICTED PRICE CHANGE

Current price	Predicted price change						
	Z_1 1.50	Z_2 1.00	Z_3 0.50	Z_4 0.00	Z_5 −0.50	Z_6 −1.00	Z_7 −1.50
dollars	*dollars/cwt*						
21.00	10.07	7.71	2.77	−2.82	−5.77	−9.75	−14.75
22.00	10.96	8.59	3.64	−1.96	−4.91	−8.89	−13.88
23.00	11.84	9.48	4.52	−1.08	−4.04	−8.02	−13.02
24.00	12.65	10.30	5.36	− .24	−3.20	−7.20	−12.21
25.00	13.52	11.16	6.22	.63	−2.32	−6.30	−11.30
26.00	14.38	12.02	7.08	1.49	−1.46	−5.44	−10.43
27.00	15.25	12.89	7.95	2.36	− .59	−4.57	− 9.57
28.00	16.15	13.78	8.83	3.22	.27	−3.71	− 8.70

able, or (b) the cost of obtaining additional information exceeded its value.

Data strategies. Under the data strategy, profit expectations are conditional on the predicted price change. Given an expected price change Z_k, the expected profit of action A_i is $\hat{\pi}|Z_k = \sum_j \lambda_{ij}P(\Theta_j|Z_k)$. $P(\Theta_j|Z_k)$ is the k^{th} column of the *a posteriori* probability distribution presented in table 13.

The expected profit, based on the *a posteriori* information, from extending the feeding period another 30 days for 900-pound steers when the current price is \$21.00 and the predicted price change is \$0.50, is calculated as follows: $\hat{\pi}_1|Z_3 = .091(12.44) + .273(7.71) + .227(2.98) + .318(-1.74) + .091(-6.47) = \$2.77$ per head. The matrix of expected profits from extending the feeding period of 900-pound steers for alternative levels of current prices and predicted price changes is presented in table 21.

Since the added profit of action A_2 (sell at current weight) is zero, the decision rules can be developed solely by looking at table 21. Action A_1 (extend the feeding period another 30 days) will be the optimal action if $\hat{\pi}_{1j} > 0$. If $\pi_{1j} \leq 0$, then A_2 is the optimal action. The feed-or-sell decision rules for 900-pound steers presented in table 22 are derived following these criteria.

The last column of table 22 shows the minimum predicted price change (β_k)[23] required to extend the feeding period at alternative levels of current price (i.e., the minimum value of Z_k for which $\hat{\pi} > 0$). The decision rule is to extend the feeding period if $Z_k \geq \beta_k$ and to sell at current weight if $Z_k < \beta_k$. For example, if the current price is \$23.00, a

TABLE 22

BAYESIAN "DATA" STRATEGIES, BASED ON *A POSTERIORI* INFORMATION: 900-POUND STEERS

Current price	"Data" strategy, given predicted price change Z_k	Minimum predicted price change required to feed (β_k)
	dollars/cwt	
21.00	Feed if $Z_k \geq$ .50; Sell otherwise	.50
22.00	Feed if $Z_k \geq$ .50; Sell otherwise	.50
23.00	Feed if $Z_k \geq$ .50; Sell otherwise	.50
24.00	Feed if $Z_k \geq$ .50; Sell otherwise	.50
25.00	Feed if $Z_k \geq$ .00; Sell otherwise	.00
26.00	Feed if $Z_k \geq$ .00; Sell otherwise	.00
27.00	Feed if $Z_k \geq$ .00; Sell otherwise	.00
28.00	Feed if $Z_k \geq -$.50; Sell otherwise	−.50

[23] β_k changes in intervals of \$0.50 because expected profits were evaluated for \$0.50 intervals in predicted price change Z_k.

TABLE 23
SUMMARY OF BAYESIAN "DATA" DECISION RULES FOR SHORT-RUN MARKETING DECISIONS—MINIMUM *PREDICTED* PRICE CHANGE REQUIRED TO EXTEND FEEDING PERIOD 30 DAYS FOR ALTERNATIVE WEIGHTS OF CATTLE AND CURRENT PRICES

Current price	Current weight of cattle								
	775–825	826–875	876–925	926–975	976–1025	1026–1075	1076–1125	1126–1175	1176–1225
dollars/cwt									
20.51–21.50	.50	.50	.50	.50	.50	1.00	.50	1.00	1.00
21.51–22.50	0	.50	.50	.50	.50	1.00	.50	.50	.50
22.51–23.50	0	.50	.50	.50	.50	1.00	.50	.50	.50
23.51–24.50	0	.50	.50	.50	.50	.50	.50	.50	.50
24.51–25.50	−.50*	.50	0	.50	.50	.50	.50	.50	.50
25.51–26.50	−.50	0	0	0	.50	.50	.50	.50	.50
26.51–27.50	−.50	0	0	0	0	.50	.50	.50	.50
27.51–28.50	−.50	−.50	−.50	0	0	.50	.50	.50	.50

* A negative value indicates that the expected profit from continued feeding is positive even if a price decline of this magnitude is expected.

predicted price change $Z_k \geq \$0.50$ is required to extend the feeding period for 900-pound steers.

The Bayesian strategies for other weights of cattle are derived by the same procedure and are summarized in table 23. The values shown in the table are the *minimum predicted price change* required to extend the feeding period for each weight category at alternative levels of current prices. For example, if we have a lot of steers averaging 1,200 pounds and current prices are \$21.35, a predicted price change $Z \geq \$1.00$ is required to extend the feeding period. If current prices are \$26.15, we can expect positive marginal profits from extending the feeding period of these steers only if the predicted price change $Z_k \geq \$0.50$.

The above decision rules are consistent with *a priori* expectations, given that the cost per pound of gain increases with the weight of the steer. The critical values of table 23 (i.e., the minimum expected price change required to extend the feeding period) tend to decrease as prices increase and increase as current weight increases.

Value of the price forecasting model. The value of the information provided by the price forecasting model is defined by equation 4 as the difference between the expected income from following the data strategy and the expected income from following the no data strategy. Consider, for example, the value of the data for 900-pound steers at alternative price levels.[24]

The no data strategy for 900-pound steers is to extend the feeding period if the current price is greater than or equal to \$23.00 and to sell at current weight otherwise (table 20). The expected income from following the no data strategy is the expected profit (table 19) of the optimal action at each price level. For example, the expected (marginal) income from following the no data strategy (i.e., sell) is zero when the price is \$22.00 and \$2.81 per head when the price is \$26.00.

The data strategy (table 23) for 900-pound steers is to extend the feeding period if (a) $P \leq \$24.00$ and $Z_k \geq \$0.50$, (b) $\$27.00 \geq P > \$24.00$ and $Z_k \geq 0$, or $P \geq \$28.00$ and $Z_k \geq -\$0.50$, and to

[24] Since each price level is considered as a separate problem, the value of the data is computed for each price level.

TABLE 24

VALUE OF INFORMATION PROVIDED BY THE PRICE FORECASTING MODEL: 900-POUND STEERS

Current price	Expected income from following each strategy		Value of data
	Data strategy	No data strategy	
dollars/cwt	*dollars/head*		
20.50–21.50	2.06	0.00	2.06
21.51–22.50	2.39	0.00	2.39
22.51–23.50	2.72	.24	2.48
23.51–24.50	3.02	1.07	1.95
24.51–25.50	3.51	1.94	1.57
25.51–26.50	4.05	2.81	1.24
26.51–27.50	4.60	3.67	.93
27.51–28.50	5.22	4.54	.68
Average	3.45	1.78	1.67

sell at current weight otherwise. Because the expected profit from extending the feeding period is conditional on the predicted price change, it is necessary to weight the expected profits for alternative values of Z_k (table 21) by the probability of predicting Z_k, $P(Z_k)$ (table 11). Thus, the expected (marginal) income, $\hat{I}$, from following the data strategy for 900 pound animals when $P = \$22.00$ is

$$\begin{aligned}\hat{I} &= .050(10.96) + .136(8.59) \\ &\quad + .185(3.64) + .226(0) \\ &\quad + .108(0) + .034(0) \\ &= \$2.39\end{aligned}$$

The expected income from following each strategy for 900-pound steers is presented in table 24. The value of the data ranges from \$2.48 when $P = \$23.00$ to \$0.69 when $P = \$28.00$. The value of the data strategy over the alternative method for 900-pound steers averages \$1.67 per head.

During the 1960–1969 period, the price of 900- to 1,100-pound Choice grade steers averaged about \$25.00. At this price level the expected (marginal) income per head over all weight groups from incorporating the information provided by the price forecasting model into the decision process exceeds by \$2.00 per head the expected (marginal) income of basing the decision solely on past price changes.

Sensitivity of model. The decision rules developed above are based on estimates of costs and returns under alternative situations for a "typical" California feedlot. This procedure raises two questions. First, how sensitive is the model to errors made in estimating costs? Second, are the decision rules applicable to feedlots whose cost relationships deviate from those specified for the typical feedlot? The second question is answered by answering the first. If the same decision rules would arise for a range of cost estimates, then we can conclude that the decision rules are applicable for feedlots with atypical cost relationships.

We can evaluate these questions by looking at the expected profit from extending the feeding period, given $Z_k = \Theta_k$ and $Z_k = [\beta_k - \$0.50]$ at alternative price levels.[25] Consider for example, the case of 900-pound steers presented in table 25. The values in column 3 indicate the expected profit from extending the feeding period of 900-pound steers given $Z_k = \beta_k$. They also indicate the amount by which estimated costs per head would have to be increased before β_k would be increased to the next highest value. The values in column 4 indicate the amount by which cost estimates would have to be decreased before β_k could be reduced to the next lower value. Taken together, these figures specify the range of cost estimates for which the decision rules are appli-

[25] Z_k is the expected price change, measured in \$0.50 increments. β_k is the minimum value of Z required to extend the feeding period (i.e., the minimum expected price change for which expected profit of continued feeding is positive).

TABLE 25
DATA FOR EVALUATING SENSITIVITY OF DECISION MODEL: 900-POUND STEERS

Current price	Minimum price change required to extend feeding period (β_k)	Expected profit from feeding at alternative values of Z_k	
		$Z_k = \beta_k$	$Z_k = \beta_k - \$0.50$
dollars/cwt		*dollars/head*	
21.00	.50	2.77	−2.82
22.00	.50	3.64	−1.96
23.00	.50	4.52	−1.08
24.00	.50	5.36	− .24
25.00	0	.63	−2.32
26.00	0	1.49	−1.46
27.00	0	2.36	− .59
28.00	−.50	.27	−3.71

cable. For example, given $P = \$21.00$, we would arrive at the same critical value, $\beta_k = \$0.50$ for estimated cost of feeding ranging from \$22.28 to \$16.69.[26]

Note that β_k changes from \$0.50 to .00 as we move from $P = \$24.00$ to \$25.00. We will refer to $P = \$25.00$ as the pivot point (the price at which β_k changes value). The location of the pivot point is sensitive to cost estimates. In the above example, if feeding costs per head had been 25 cents lower than the estimate shown in table 17, the pivot point would have been $P = \$24.00$. On the other hand, if feeding costs would have been 64 cents higher than the estimate used, the pivot point would have been $P = \$26.00$. Thus, if the decision rules are being used for a feedlot whose costs are above those estimated here, the pivot point could be moved from $P =$ \$25.00 to $P = \$26.00$. Similarly, the pivot point could be moved to $P =$ \$24.00 if the cost structure is below that used above.

Based on this analysis, the decision rules are rather insensitive to variations of feeding costs and are applicable to a wide range of feedlot situations. However, as noted above, a slight modification of the pivot point might be in order if costs of feeding deviate from the estimates used here. Evaluation of similar data for other weight groups supports these conclusions.

Part of the stability of the decision rules is built into the model by using \$0.50 increments in price changes. Smaller price change increments would make the model more sensitive to variation in per head feeding cost. The decision rules summarized in table 23 are conservative since there may be predicted price changes less than β_k for which the expected marginal profit from continued feeding is still positive. All values of predicted price changes between $(\beta_k - \$0.50)$ and β_k were not evaluated since our probability distributions were geared to \$0.50 increments in price change. Therefore, β_k may be an overestimate of the breakeven predicted price (i.e., the predicted price change for which $\hat{\pi} = 0$).

Define the break-even price change, β_k', as $\beta_k' = \beta_k - \hat{\pi}/W_2$, where $\hat{\pi}$ is the expected per head marginal profit if $Z_k = \beta_k$ and W_2 is the weight of the steer (100 pounds) at the end of the extended feeding period. There is a lower bound on β_k', defined by the lower bound of the price change interval for which $\beta_k = Z_k$ is the representative value.[27] Thus, if $\beta_k = Z_k = \$0.50$, the lower bound of β_k' is \$0.26 since the probabilities used to calculate the expected income for $Z_k = \beta_k$ are applicable only to price changes in the range $\$0.26 \leq Z_k \leq$ \$0.75. Consequently, a different set of probabilities has to be used to calculate expected income if $Z_k < \$0.26$. There-

[26] Estimated cost of feeding 900-pound steers another 30 days is \$19.51. \$19.51 + \$2.77 = \$22.28 and \$19.51 − \$2.82 = \$16.69.

[27] The intervals for which Z_k is the representative price are the same as the intervals for Θ_j, $j = k$ as shown in table 7.

TABLE 26

EXPECTED PROFIT, $\hat{\pi}$, (DOLLARS PER HEAD) FROM EXTENDING THE FEEDING PERIOD ONE MONTH: GIVEN THAT $Z_k = \beta_k$

Current price of slaughter steers	Weight of cattle on feed (pounds)								
	775–825	826–875	876–925	926–975	976–1,025	1,026–1,075	1,076–1,125	1,126–1,175	1,176–1,225
dollars/cwt	*dollars*								
20.50–21.50	4.12	.26	2.77	2.16	1.63	3.18	.41	6.24	6.01
21.51–22.50	.21	1.48	3.64	3.02	2.47	4.14	1.21	.69	.16
22.51–23.50	1.04	2.66	4.52	3.89	3.32	5.12	2.00	1.45	.92
23.51–24.50	1.92	3.89	5.36	4.71	4.13	.26	2.86	2.30	1.77
24.51–25.50	.27	5.10	.63	5.56	4.97	1.27	3.68	3.11	2.57
25.51–26.50	1.11	.98	1.49	.51	5.80	2.27	4.49	3.91	3.35
26.51–27.50	1.98	2.18	2.36	1.36	.45	3.26	5.34	4.75	4.18
27.51–28.50	2.81	.56	.27	2.22	1.29	4.27	6.16	5.55	4.98

fore it is necessary to set a lower bound on β_k' equal to the lower bound on $Z_k = \beta_k$.

The decision rules can be modified with the information about β_k'. For example, if the decision rules specify $\beta_k =$ \$0.50 and it turns out that $\beta_k' =$ \$0.30, then if $Z_k >$ \$0.30, the expected profits from extending the feeding period are positive. In this case, the cattle feeder might want to extend the feeding period even though a sell decision is generated by the decision rules of Model I.

The above modification of the decision rules emphasizes the accuracy of the estimated cost of feeding another 30 days. This modification is applicable only if the cost per head of feeding another 30 days in the feedlot for which the decision is being made (C') is identical with the estimated cost (C) used to derive the decision rules. If $C' \neq C$, then the appropriate adjustment can be calculated as $\beta_k'' = \beta_k - [\hat{\pi} - (C' - C)]/W_2$. Thus, for feedlots where $C' > C$, β_k'' will be less than β_k' and if $C' < C$, then β_k'' will be greater than β_k'. The same set of lower bounds applies for β_k'' as does for β_k'. The values of $\hat{\pi}$ needed to make these adjustments are shown in table 26.

The expected income of the strategies could be increased if decisions were based on a comparison of Z_k with β_k' rather than β_k. However, these decision rules would be incorrect for feedlots with per head feeding costs above those used in the study and would still be conservative for feedlots whose costs are below the estimates used. Therefore if a more precise decision rule is desired, the values of β_k in table 23 would have to be replaced by the values of β_k'' which are applicable to the feedlot in question.

Feeding decisions for two months—Model II

The purpose of Model II is to develop a set of decision rules utilizing 60-day price predictions with which to reevaluate a sell decision generated by Model I for steers weighing less than 1,000 pounds.

Model II uses the same theoretical framework and similar input data as Model I with a reduced number of states of nature (price changes) from seven to three. The states of nature used in Model II were shown previously, and the *a posteriori* probability distributions for 60-day price projections are presented in table 12.

The weight of cattle at the end of the extended feeding period, cost of feeding, and proportion grading Choice at alternative weights are presented in table 27.

TABLE 27

INPUT DATA NEEDED TO DERIVE DECISION RULES FOR MODEL II

Current weight	Weight at end of extended feeding period	Cost of feeding	Proportion of cattle grading choice at	
	60-day	60 days	Current weight	End of 60-day extension
pounds		*dollars/head*		
800	967	37.67	.263	.508
850	1,015	38.86	.360	.551
900	1,064	40.14	.432	.590
950	1,112	41.31	.490	.624

Given this information along with the *a posteriori* distributions, the decision rules are calculated by the same procedure as used for Model I.

The resulting decision rules are summarized in table 28. The values shown in the table are the minimum expected price change (β_k) required to extend the feeding period. If the predicted price changes are below the values in the table the cattle are sold. For example, given a lot of steers averaging 850 pounds and the current price of $23.20, a predicted price change of $1.00 over the next 60 days is required to override a sell decision in Model I (table 28). If the predicted price change over the 60-day period is less than $1.00, the decision is to sell.

Feeder cattle purchase decisions—Model III

Model III develops decision criteria for purchasing feeder cattle. The model specifies that the cattle feeder purchases 600-pound Good grade feeder steers, based on the expected value of the steer at the end of a 150-day feeding period.[28] The decision rules compare the predicted price of Choice grade 900- to 1,100-pound slaughter steers five months hence with the break-even price of slaughter cattle, given the current price of feeder cattle. The break-even price (P^S) is the price of a 900- to 1,100-pound Choice grade slaughter steer *required* at the end of the feeding period to equate gross value of the slaughter steer with the cost of the feeder steer plus cost of feeding. Fed cattle weighing 1,015 pounds on the average will grade 55.1 per cent Choice and 44.9 per cent Good (equation 11). Given the relationship between the price of Good grade slaughter steers and that of the Choice grade animals (table 5), the required breakeven gross value of the steer at the end of the feeding period can be defined as: $GV = 10.15\ [.551P^S + .449(.848 + .922P^S)]$, or $GV = 3.865 + 9.794P^S$.

The cost of feeding over the 150-day period has two components, feed costs, and nonfeed costs. Feed costs per pound of gain average 19.31 cents over the 600- to 1,000-pound feeding interval (table 15). Thus, the feed costs over the 150-day feeding period are (415)(.1931) = $80.14. Nonfeed costs total (150)(.0646)

TABLE 28

SUMMARY OF DECISION RULES FOR MODEL II—MINIMUM PREDICTED PRICE CHANGE OVER NEXT 60 DAYS REQUIRED TO OVERRIDE A SELL DECISION FROM MODEL I

Current price	Current weight of steers			
	775–825	826–875	876–925	926–975
dollars/cwt				
20.51–21.50....	1.00	1.00	1.00	1.00
21.51–22.50....	1.00	1.00	1.00	1.00
22.51–23.50....	1.00	1.00	1.00	1.00
23.51–24.50....	0	1.00	1.00	1.00
24.51–25.50....	0	0	0	1.00
25.51–26.50....	0	0	0	1.00
26.51–27.50....	−1.00*	0	0	1.00
27.51–28.50....	−1.00	−1.00	0	0

* Negative values indicate that continued feeding would be profitable even if prices declined by $1.00/cwt.

[28] This corresponds to the "normal" feeding period of 600–1,000 pounds used previously. If the cattle gain 2.77 pounds per head per day (table 15), they will weigh 1,015 pounds at the end of the 150-day feeding period.

= \$9.69 per head. Therefore, total cost of feeding a steer for 150 days is estimated to be \$89.83.

The break-even price of slaughter cattle (P^S) can be calculated for alternative levels of current feeder cattle prices (P^F) as shown by equation (13).

$$10.15\,[.551P^S + .449(.848 + .922P^S)] = 6.0P^F = 89.83$$

$$3.865 + 9.794P^S = 6.0P^F + 89.83 \quad (13)$$

$$P^S = 8.777 + .613P^F$$

TABLE 29

POSSIBLE DECISION RULES FOR PURCHASING FEEDER CATTLE BASED ON COMPARISON OF EXPECTED PRICE, $\hat{P}_5$, AND BREAK-EVEN PRICE P^S

Expected profit margin $\hat{P}_5 - P^S = \hat{M}$	Proportion of vacant capacity to be purchased	Probability of a loss Prob $\{P_5 - P^S < 0 \mid \hat{M}\}$
dollars/cwt		*per cent*
$\hat{M} \leq .25$	No purchase	—
$.26 \leq \hat{M} \leq .75$	.15	< 48
$.76 \leq \hat{M} \leq 1.25$	.30	< 30
$1.26 \leq \hat{M} \leq 1.75$	.45	< 20
$1.76 \leq \hat{M} \leq 2.25$	.60	< 10
$2.26 \leq \hat{M}$	.75	< 5

If the current price of feeder cattle[29] is \$24.00 per hundred pounds, the break-even price of Choice grade slaughter steers five months hence is $P^S = 8.777 + .613(24.00) = 23.49$.[30]

For a given level of current feeder cattle prices, the cattle feeder will purchase feeder cattle only if the predicted price of 900- to 1,100-pound Choice grade slaughter steers is above the break-even price. That is, feeder cattle will be purchased only if the expected profit margin $\hat{M} = \hat{P}_5 - P^S > 0$, where $\hat{P}_5$ is the predicted price of slaughter cattle five months hence.

The cattle feeder is concerned about the risk of loss involved as well as the expected profit margin. That is, given an expected profit margin, $\hat{M} = (\hat{P}_5 - P^S)$, he is concerned about the probability that $P_5 - P^S < 0$, where P_5 is the realized price at the end of the feeding period.

The price predictor developed earlier by least-squares estimation assumes that the error term associated with the equation is normally distributed with mean 0 and variance σ^2. On this basis, the variable $P_5 - \hat{P}_5/S_{\hat{P}_5}$ follows a t-distribution with $n - k$ degrees of freedom (Goldberger, 1964, p. 179),[31] where S_{P_5} is the standard error of the estimate. Consequently, the prediction errors $(P_5 - \hat{P}_5)$ are symmetric about zero. By restricting purchases to situations $\hat{M} = (\hat{P}_5 - P^S) > 0$, the probability of a loss occurring (i.e., Prob $\{(P_5 - P^S) < 0\}$) never exceeds 50 per cent. Moreover, as $\hat{M}$ increases, the probability of a loss decreases.[32]

The decision rules developed below are based on three hypotheses. First, some minimum expected profit margin, $\hat{M} > 0$, is necessary to induce the cattle feeder to place cattle on feed, depending on the cattle feeder's aversion to risk. Second, the number of cattle placed on feed at any given time is a function of the expected profit margin, given the vacant capacity in the feedlot. For example, more cattle would be placed on feed if $\hat{M} = \$3.00$ than if $\hat{M} = \$0.50$. Third, placements of cattle on feed in any given month will be less than 100

[29] The price referred to here is the price of 550- to 750-pound Good grade feeder steers.

[30] The break-even margin is calculated as the break-even price minus the current price of feeder cattle, i.e., \$23.49 − \$24.00 = \$−0.51.

[31] n is the number of observations used to estimate the equation and k = the number of parameters estimated in the equation; $n = 38$ and $k = 11$ for the first two five-month price prediction equations shown in table 11 and $n = 39$ and $k = 11$ for the third equation.

per cent of *vacant* capacity, and there is a limit to the number of feeder cattle that will be purchased in any one month (e.g., 20 per cent of total feedlot capacity).

The decision rules presented in table 29 are admittedly arbitrary and will vary from feeder to feeder depending on this aversion to risk and the operating characteristics of the feedlot. The minimum expected profit margin required for purchasing feeder cattle (i.e., \$0.25) and the proportion of vacant capacity to be filled at alternative values of $\hat{M}$ are not based on any particular optimizing criteria; rather, they are presented as an example of the decision rules that can be developed. The value of these parameters can readily be changed to conform with the desires of a given cattle feeder. The only calculations required to make these changes are the probabilities of making a loss (i.e., Prob $\{P_5 - P^S < 0|\hat{P}_5 - \hat{M} = P^S\}$).

The probablity of a loss given a predicted profit margin, $\hat{M} = \hat{P}_5 - P^S$, is calculated as follows:

$$\text{Prob } \{P_5 - P^S < 0|\hat{P}_5 - \hat{M} = P^S\}$$

$$= \text{Prob } \{P_5 - \hat{P}_5 < -\hat{M}\}$$

$$= \text{Prob } \frac{P_5 - \hat{P}_5}{S_{\hat{P}_5}} < \frac{-\hat{M}}{S_{P_5}}$$

$$= \text{Prob } t < \frac{-\hat{M}}{S_{\hat{P}_5}}$$

The probability that $t < -M/S_{P5}$ can be found from a table of values for the t-distribution using $(n - k)$ degrees of freedom.

Since there are separate equations for making five-month forecasts for the three months of the quarter, there is one value of $S_{\hat{P}5}$(\$1.28) the first month of the quarter, another $S_{\hat{P}5}$(\$1.38) the second month, and still another $S_{\hat{P}5}$(\$1.55) for the third month of the quarter. The probability of a loss, given $P_5 - \hat{M} = P^S$, differs only slightly between these values of $S_{\hat{P}5}$ but increases as $S_{\hat{P}5}$ increases.

The break-even slaughter cattle price equation can be adjusted by replacing the estimated feeding cost (C) with the actual cost (C') of the particular feedlot. The minimum expected profit margin $\hat{M}$ required for purchasing feeder cattle can be specified by the cattle feeder depending on his aversion to risk. The quantity purchased criteria in column 2 of table 29 also will depend on the operating characteristics of the feedlot.

Once the cattle feeder has specified the appropriate parameters, only a few simple calculations are needed to make purchasing decisions. First, calculate the break-even price of slaughter cattle five months ahead, given the current price of feeder cattle. Second, project the price of slaughter cattle five months ahead. Third, calculate the expected profit margin $\hat{M} = \hat{P}_5 - P^S$. And, fourth, using the decision parameters specified by the feedlot operator, de-

[32] This is shown as follows:

$$\text{Prob } \{P_5 < P^S|\hat{P}_5 - \hat{M} = P^S\} = \text{Prob } \{P_5 < \hat{P}_5 - \hat{M}\}$$
$$= \text{Prob } \{P_5 - \hat{P}_5 < -\hat{M}\}$$
$$= \text{Prob } \frac{P_5 - \hat{P}_5}{S_{P_5}} < -\frac{\hat{M}}{S_{\hat{P}_5}}$$
$$= \text{Prob } t < -\frac{M}{S_{\hat{P}_5}}$$

Given that $\hat{M}_1 > \hat{M}_2$, we know,

$$\text{Prob } \left\{t < -\frac{\hat{M}_1}{S_{\hat{P}_5}}\right\} < \text{Prob } \left\{t < -\frac{\hat{M}_2}{S_{\hat{P}_5}}\right\}.$$

termine the number of head to be purchased. The probability of loss occurring (i.e., $P_5 < P^S$) given $\widehat{M}$ can be calculated as shown above.

Purchase and sales decisions—Model IV

Uncertainty about future prices precludes precise planning as to the pattern of placements and marketings into the future; however, some tentative plans are needed about future sales and purchases in order to assess future needs for capital and feed. Model IV incorporates the decision rules developed, along with the results of the price forecasting model, into a six-month planning model. The resulting information can only be considered as *tentative* and is not precise enough to serve as a basis for definite decisions, but does provide an indication of the possible pattern of sales and purchases over the next six months and thus, may be useful in forward planning.

Model IV simulates the buying and selling activities of the feedlot over the next six months, given the current inventories of cattle in the feedlot and the capacity of that lot. This simulation is based on predictions of slaughter cattle prices up to 11 months ahead and feeder cattle prices up to five months ahead. The decision rules developed are applied sequentially to the marketing and purchasing decisions that would confront the cattle feeder over the next six months *if* the predicted prices were in fact the true prices. The resulting pattern of purchases and sales serves as a tentative identification of the set of decisions that *may* evolve during the next six months.

Given the above information, the model (1) uses the results of Models I and II to calculate sales for the first month (i.e., decide which lots currently on feed should be sold and which lots continue feeding), (2) calculates vacant capacity as total capacity minus inventory less sales, (3) decides how many feeder cattle will be purchased in first month, based on decision rules and predicted prices in Model III; and (4) calculates inventory of cattle on feed by weight groups at the end of the first period. The inventories then become beginning inventories for period 2, and the four steps are repeated for the second period. This process is continued through six months. At the end of the sixth sequence, one can observe the expected pattern of sales and purchases.

As an example of how Model IV is used, consider the following tentative plan of sales and purchases over a six-month period for a 10,000-head capacity feedlot with the following current inventory of cattle on feed:

Weight[33] *pounds*	*Number currently in feedlot*
$W_1 =$ 600	500
$W_2 =$ 684	1,500
$W_3 =$ 767	500
$W_4 =$ 850	1,500
$W_5 =$ 933	500
$W_6 =$ 1,015	1,500
$W_7 =$ 1,096	500
$W_8 =$ 1,174	1,000
$W_9 =$ 1,250	500
Total	8,000

In addition, we have the following price information and predictions.[34]

[33] All cattle are assumed to be placed on feed weighing 600 pounds. The daily gain information presented in table 15 defines the above weights for 30-day feeding intervals. That is, cattle on feed 60 days will weigh 767 pounds, and so on.

[34] The i subscript on P and PP refers to the length of projection. Thus, P_5 is the projected slaughter price five months hence: $i = 0$ indicates current prices.

Price of 900- to 1,100-pound Choice grade steers		*Price of 550- to 750-pound Good grade feeder steers*
	dollars per hundredweight	
$P_0 = 24.95$	$P_6 = 23.93$	$PP_0 = 23.05$
$P_1 = 25.78$	$P_7 = 25.74$	$PP_1 = 23.50$
$P_2 = 25.03$	$P_8 = 25.09$	$PP_2 = 23.50$
$P_3 = 24.44$	$P_9 = 24.28$	$PP_3 = 23.50$
$P_4 = 25.18$	$P_{10} = 25.05$	$PP_4 = 23.38$
$P_5 = 24.23$	$P_{11} = 24.30$	$PP_5 = 22.26$

The first step in the simulation is to determine the number of cattle that will be sold in the first month. In accordance with the decision rules of Model I, cattle weighing less than 775 pounds (i.e., the cattle in weight groups W_1, W_2, and W_3) will be continued on feed and cattle weighing more than 1,225 pounds (weight group W_9) will be sold. The decision to keep or sell the other cattle is based on the decision rules summarized in table 24. The predicted price change $Z_k = P_1 - P_0 = \$0.83$. Since this exceeds the minimum value specified in table 23 for each weight group given the current price is \$24.95, the decision in period one is to extend the feeding period for the cattle in weight groups W_4, W_5, W_6, W_7, and W_8. Thus, sales in period one equal 500 head from weight group W_9.

The second step is to calculate vacant capacity in the feedlot after the sales indicated above. Sales in period one are 500 head. Thus vacant capacity = 10,000 − (8,000 − 500) = 2,500 head.

The third step is to calculate the number of cattle to purchase in period one based on the purchasing decision rules presented in table 29. The current price of feeder steers in the first period is \$23.05 thus the break-even price of slaughter cattle $P^S = 8.777 + .613(23.05) = \$22.91$. The expected profit margin $\widehat{M} = \hat{P}_5 - P^S = 24.23 - 22.91 = \$1.32$. The decision rules in table 29 indicate that for $1.26 \leq \widehat{M} \leq 1.75$ purchases of feeder cattle will be 0.45 (vacant capacity) = 0.45(2,500) = 1,125 head.

We can now summarize the sales and purchase transactions for period one and calculate the inventories for the beginning of period two. Five hundred head of 1,250-pound steers were sold in period one and 1,125 head of 600-pound steers were purchased. Retained cattle on feed from the beginning of the period will advance one weight group by the beginning of the second period (neglecting death loss). Thus, the beginning inventory for the second period is as follows:

$W_1 = 1{,}125$	$W_4 = 500$	$W_7 = 1{,}500$
$W_2 = 500$	$W_5 = 1{,}500$	$W_8 = 500$
$W_3 = 1{,}500$	$W_6 = 500$	$W_9 = 1{,}000$

The predicted price change between the first and second period is $Z = \$25.03 - \$25.78 = \$-0.75$.[35] Thus, based on the decision rules summarized in table 23, the decision is to sell all cattle in weight groups W_4, W_5, W_6, W_7, and W_8. As in period one, cattle in weight groups W_1, W_2, and W_3 will be kept and cattle in weight group W_9 will be sold. The cattle in weight groups W_4 and W_5, however, are less than 1,000 pounds; therefore, it is necessary to

[35] P_1 takes on the role of current price since the decision rules are being evaluated at the beginning of period two (end of period one).

TABLE 30

SUMMARY OF SIMULATING MARKETING AND PURCHASING DECISIONS SIX MONTHS INTO THE FUTURE FOR HYPOTHETICAL EXAMPLE

	Period = 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Sales	—	500	5,500	1,500	0	500	0
Purchases	—	1,125	1,031	2,000	2,000	1,153	2,000
Weight group	Inventories						
W_1	500	1,125	1,031	2,000	2,000	1,153	2,000
W_2	1,500	500	1,125	1,031	2,000	2,000	1,153
W_3	500	1,500	500	1,125	1,031	2,000	2,000
W_4	1,500	500	1,500	500	1,125	1,031	2,000
W_5	500	1,500	0	0	500	1,125	1,031
W_6	1,500	500	0	0	0	0	1,125
W_7	500	1,500	0	0	0	0	0
W_8	1,000	500	0	0	0	0	0
W_9	500	1,000	0	0	0	0	0
Total	8,000	8,625	4,156	6,656	7,309	7,309	9,309

evaluate the expected returns from extending the feeding period 60 days based on rules developed in Model II.

The predicted price change over 60 days, is \$−1.34. The decision rules summarized in table 28 indicate that for $P_1 = 25.78$ and cattle weighing 850 and 933 pounds, a predicted price change of zero or greater is required to extend the feeding period beyond 30 days for the 850-pound animals and \$1.00 or more for the 933 pound animals. Therefore, the sell decision generated above is appropriate, and sales for period two are 5,500 head.

Vacant capacity for the second period is $10{,}000 - (8{,}625 - 5{,}500) = 6{,}875$.

The current price of feeder cattle in period two is \$23.50. Therefore, the break-even price is $P^S = 8.777 + .613(23.50) = \$23.18$. The predicted slaughter price five months hence is \$23.93, thus $\widehat{M} = \$0.75$. The decision rules in table 29 indicate that when $0.25 \leq \widehat{M} \leq 0.75$ purchases of feeder cattle = 0.15 (vacant capacity) = .15(6,875) = 1,031 head.

Thus sales = 5,500 head in period two and purchases = 1,031 head. The inventories at the beginning of period three are as follows:

$$\begin{array}{lll} W_1 = 1{,}031 & W_4 = 1{,}500 & W_7 = 0 \\ W_2 = 1{,}125 & W_5 = 0 & W_8 = 0 \\ W_3 = 500 & W_6 = 0 & W_9 = 0 \end{array}$$

The procedure used to simulate periods one and two is continued through periods three, four, five, and six. The results of Model IV for the above problem are summarized in table 30.

Model IV is a planning model only in the sense that it may provide some insight as to future developments. The simulation procedure of Model IV *does not develop* a set of decision rules by which future marketing and purchasing decisions can be made. Rather, it provides only *an indication* of the sales and purchasing pattern that may evolve over the next six months. In order to use the model for a particular feedlot, one would only have to change the capacity constraint to the appropriate level and replace the feeder cattle purchasing criteria with those specified by the cattle feeder.

SUMMARY AND CONCLUSIONS

The purpose of this study has been to present an application of statistical decision theory to managerial decision-making in cattle feedlot procurement and marketing.

The analysis indicates the value of more sophisticated decision-making mechanisms to business executives. Additional information through price forecasting or other experiments can help increase the expected returns to a firm and reduce the variance of outcomes from such decisions. If a firm is vulnerable to large losses, reduction in risk and uncertainty may become a major goal in itself in decision making. Thus, a means of incorporating uncertainty directly into the decision process should prove useful.

In this analysis, the primary focus was on the uncertainty associated with imperfect knowledge about prices of fed cattle. An attempt was made to reduce the uncertainty surrounding marketing and procurement decisions by utilizing a price forecasting model in the decision process. The results of the marketing model indicate that use of such an explicit price predicting method would add about $2 per head to the expected revenues as compared with a more naive marketing strategy.

The price forecasting model projects average monthly prices of 900- to 1,100-pound Choice grade slaughter steers at El Centro, California, up to 12 months ahead. Similar projections are made for prices of 550- to 750-pound Good grade feeder steers at Stockton, California, up to six months ahead. The model is not a perfect price predictor, but it does provide a means whereby the cattle feeder can utilize information about future price changes in making decisions. The conditional probabilities of actual price changes attaining certain magnitudes given the predicted price change permits the feeder to make decisions as to whether to continue feeding animals currently in the feedlot an additional month or two or whether to sell them at their current weight.

While the models in this study illustrate the use of statistical decision theory, they are simplified to some degree and may require adjustment before actual application to a given feedlot operation. For example, factors such as weight gain, feed conversion, or change in grade affect costs or revenues. These factors all have stochastic attributes which have been considered negligible by setting their levels at their expected values. To the extent that these variables assume values more detrimental to profits than postulated here, the net effect of this type of uncertainty would be reflected in a greater needed expected price increase than suggested by the model before reaching the decision to feed another month. A sensitivity check of the models, however, shows that similar results would be obtained over a fairly wide range of costs.

Another limitation may be found in the one-month time period allocated to the decision process. Actual decision periods are continuous, not discrete. However, the nature of the data available for the price forecasting model restricted the time period to monthly intervals. This limitation, however, is not critical to the nature of the decision model, since the same procedure could be used for any length of time.

Because the replacement component of decision making has not been included, the models are more generally applicable to feedlots operating with excess capacity and available capital for additional investment in feeder cattle. Similarly, the models can be used by small feedlots which feed only a single lot per year.

Probably the greatest area for potential improvement, however, is in the price forecasting system. Modification of the model could include explicit recognition of the effects of prices or quantities of other meat animals and of changes in income. A tendency toward lighter marketing weights observed in 1967–1968 suggests that the average weight of cattle sold from feedlots may be an important variable. The model used in this study is based only on numbers of cattle marketed and lagged prices, and therefore, assumes that the average weight of fed cattle marketed will remain at the 1960–1969 average. Consequently, if average weights decline (increase) the model will over (under) estimate the supply of beef actually coming on the market and thus under (over) estimate prices.

The problem of making the modifications necessary to adjust for the above factors will be the ability to predict significant variations in these variables and still maintain the simplicity of the model. A balance will have to be attained between the value of the information gained as reflected in improved prediction accuracy and the cost of the added information in terms of collection costs and increased complexity.

No attempt was made in this study to compare the decision rules developed with other decision rules. Comparison of expected incomes arising from the decision rules developed in the study with decision rules used by actual cattle feeders would be of particular interest, provided one could explicitly state the decision rules used by a cattle feeder. The comparison of average income and income variability could be accomplished by simulating the operations of the feedlot over some period of time under these alternative decision rules. For example, one could simulate the operations and resulting income flows using (a) the "data" strategies, (b) the "no data" strategies, (c) perfect knowledge, and (d) other decision rules available to feedlot operators such as marketing at 1,000 pounds regardless of current prices. Information gained from this simulation would provide a basis for selecting among alternative decision rules by considering income variability as well as expected income. The survey and quantification of decision rules used by cattle feeders constitutes a study in itself and thus was not attempted here.

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